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REPRESENTATION ON QUASI PSEUDO GPW- INJECTIVE RINGS AND MODULES

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Abstract :-

In this paper generalize the notion of quasi , pseudo injective module and quasi pseudo GP-w- injective rings and module. Hence introduce and study more properties of pseudo-w injective module, pseudo GP-w- injective module and the endomorphism rings of pseudo –w – injective modules.

Introduction :- Throughout the paper R is an associative ring with identity $1\neq0$ and all modules are unitary R-modules. We write MR (resp. RM) to indicate that M is a right or left R-module. Let J (resp. M_r , S_r) be the Jacobson radical resp. the right singular ideal the right socle of R and $E(M_R)$ the injective hull of M_R if X is a subset of R the right or left annihilator of X in R is denoted by $r_R(X)$ (resp. $l_R(X)$) or simply r(X) (resp.l (X)). If N is a sub module of M resp. proper right $N \leq^e M$, N<<M , N≤ ⊕M and N ≤max M to indicate a direct summand and a maximal submodule of M resp. sub module of M is essential in M. A Module M is finite dimensional if E(M) is a finite direct sum of indecomposable sub module. A right R module N is called M-generated if there exist End M(I0 →N for some index set I. If the set I is finite then N is called finitely M-generated. In particular N is called M-cyclic if it is isomorphic to M/L for some sub module L of M. Hence any M cyclic sub module X of M can be considered as the image of End of M.

By Nicholson A ring R is called w-injective if every R-Hom. From Principally ring ideal of R to R is a left multiplication. In[18] sanhet.al transferred this notion to modules . A right R-module N is called M-w- injective if every Hom. From an M cyclic sub module N can be extended to one from M to N. A right R module M is called quasi-w- injective if M is M-w-injective. Quasi-w –injective modules were under terminology of semi-injective modules but there are no details. Following [13] a module M is called weak quasi injective if every Hom. From a cyclic sub module of M to M can be extended to an End. Of M. since M is cyclic sub module of M needs not to be cyclic the notion of quasi –w- injective module is different.

As a generation of injective module given by singh and jain for notion of pseudo M-injective module A right R-module N is called pseudo M- injective module if for every sub module G of M any monomorphism f : G→N can be extended to a Hom. M→N . A right R- module N is called pseudo injective if N is pseudo N-injective.

In 2009 notion of pseudo M-w- injective modules introduced and studied the End. Rings of quasi pseudo –w injective modules . A right R- module N is called pseudo M-w-injective if every monomorphism from an M cyclic sub module of M to N can be extended to a Hom. From M to N or equivalently for any homomorphism f ∈ End(M) , every monomorphism from f(M) to N can be extended to a Hom. From M to N.

A module M is called quasi pseudo –w injective if M is pseudo m-w injective. Following [8] , a right R module M is said to be generalized principally w injective (short GP w injective) if for any 0∈x ∈ R there exist an n ∈N such that $x^n\neq 0$ and any R-Hom. From x^nR into M can be extended to one from R_R to M.

A ring R is called GP-w injective if R_R is GP w injective. The concept of gpw injective module was introduced to study the class of von Neumann regular rings , V rings , self injective rings and their generalization. In particular gave some characterization of GP-w-injective ring with special chain conditions.

A ring R – module N is called for pseudo M- GP w injective of for each Hom. 0 ≠ f ∈ End(M) there exist n ∈N such that fn ≠ 0 *and every monomorphism* from fn (M) to N can be extended to a Hom. From M to N (17). A module M is called qp.gp-w injective if M is pseudo M- GP w injective. A ring R is called right pseudo GP-w- injective if R_R is quasi pseudo GP w- injective. In this paper we continue studying more properties of pseudo w- injective module ,pseudo –GP-w injective modules and the End. Rings of pseudo –w- injective modules.

[1] **Theorem 1.1 :-** Let M, N be right R-modules then following conditions are equivalent :

a) N is pseudo M- GPw –injective

b) For each $0 \neq g \in$ End (M) there exist $n \in N$ such that $g^n \neq 0$ and { $h \in$ Hom. (M,N) │ ker h = ker g^n} ⊆ Hom. (M, N)g^n

c) For each $0 \neq g \in$ End (M) there exist $n \in N$ such that $g^n \neq 0$ and { $h \in$ Hom. (M,N) │ ker h = ker g^n} = { $h \in$ Hom. (M,N) │ ker h ∩ Im g^n = 0} g^n

Proof :- (a)→(b) Suppose that $0 \neq g \in$ End (M). since N is pseudo M-GPw- injective there exist $n \in N$ such that $g^n \neq 0$ and every monomorphism from gn (M) to N can be extended to a Hom. From (M,N) such that ker h = ker gn. We consider Hom. φ : g^n (M)→N via φ (g^n(M)) = f(M) for all m ∈M . It is easy to see that . φ is a monomorphism. By assumption there exist Hom.

K : M→N such that k l = γ where l is the inclusion map from g^n (M) →M⟹ h = gn ∈ Hom. (M,N) g^n

(b)→(c) it is clear that { $h \in$ Hom. (M,N) │ ker h ∩ Im g^n = 0} g^n ⊆ { $h \in$ Hom.(M,N)│ker h = ker g^n}

Let $s \in$ Hom. (M,N) such that ker s = kergn then by g there exist a Hom. (M,N) such that ker s = ker g^n then by (b) there exist Hom. K : M→N such that s=kg^n. It follows that ker k∩ Im g^n = 0 hence s ∈ { $h \in$ Hom.(M,N)│ker h ∩ Im g^n=0 } g^n

(c)→(a) for each $0 \neq g \in$ End (M) by [3] there exist $n \in N$ such that $g^n \neq 0$ and

{ h ∈ Hom. (M,N)│ker h = ker g^n} = { h ∈ Hom. (M,N)│ker h ∩ Im g^n = 0 } g^n

Assume that Ψ : g^n(M)→N is monomorphism then ker (Ψgn) = ker gn hence there is $s \in$ Hom.(M,N) such that Ψgn=sgn. It gives sl=ϕ there l is the inclusion map proving that N is pseudo M- GPw –injective.

Corollary 1.2: - Let M be right R-module and G= End (M) the following conditions are equivalent:

(1) M is quasi pseudo GP-w-injective.
(2) For each $0 \neq g \in G$ here exist $n \in N$ such that gn≠0 and {f ∈G│ker f = ker gn } ⊆ Gg^n
(3) For each $0 \neq g \in G$ there exist $n \in N$ such that gn≠ 0 and {f ∈G│ker f = ker gn }={ f ∈ G│ker f ∩ Im g^n = 0 }

Proposition 1.4 :- Let N be pseudo M-w injective then for any elements g, f ∈ End (M). we have

{ $\in Hom.(M,N)$ │ Ker β ∩ Im g = Ker f ∩ Im g } = { $\in Hom.(M,N)$ │Ker γ ∩ Im (gf) = 0 }f +{ $\delta \in Hom.(M,N)$ │ δg = 0 }

Proof :- Let A = { $\beta \in Hom.(M,N)$ │ Ker β ∩ Im g = Ker f ∩ Im g }

B = { $\gamma \in Hom.(M,N)$ │Ker γ ∩ Im (gf) = 0 }

C = { $\delta \in Hom.(M,N)$ │ δg = 0 }

It is easy to see Bf+C ⊑ A Conversely let $\beta \in Hom.(M,N)$ such that Ker β ∩ Im g = Ker β ∩ Im g(B ∈ A) It follows that Ker (gf) = Ker (βg) By corollary 1.3 there exist $\gamma \in B$ such that βg = γfg or ($\beta - \gamma$f)g = 0 Means ($\beta - \gamma$f) ∈ C which implies that β ∈ Bf+C.

Proposition 1.5 :- If M = $M_1 \oplus M_2$ is quasi pseudo-w-injective then M_1 is M_2 –w-injective.

Proof :- Let M = $M_1 \oplus M_2$ be quasi pseudo –w-injective and g$\in$ End (M_2). Let f : g (M_2)→M_1 be a Homomorphism. Consider p : g (M_2)→M p(a)= f(a)+a for all a$\in$ g(M_2) then g is a monomorphism. By proposition 1.3 M is pseudo M2-w-injective where g extends to a homomorphism $\bar{p}$: M_2→M let $\prod$: M→M_1 be the canonical projection then $\prod \bar{p}$: M_2→M extends f. thus M_1 is M2-w- injective as required.

On quasi pseudo GP w- injective rings and modules

Some characterization of quasi pseudo w- injective modules

Theorem 2.1 :- The following conditions are equivalent for module M with S = End (M)

1. M is quasi pseudo w- injective
2. If Ker f = Ker g with f , g ∈ S= End (M) then sf=sg
3. If f ∈ S = End(M) and α, β : f(M)→M is monomorphism then $\alpha = S\beta$ for some s∈S

Proof :- (1)→(2) By Cor. 1.3

(2)→(3) Assume that 0≠f∈S satisfies (2) let α, β : f(M)→M be monomorphism then Ker(αf)=Ker(βf). By Our assumption there exist s∈ S and $\emptyset : s(M) \to M$ be a monomorphism. Let l : s(M)→M be the inclusion by (3) there exist $\overline{\emptyset}$: ∈ S such that $\emptyset = \overline{\emptyset}$ l showing that $\overline{\emptyset}$ extends ∅ thus M is quasi pseudo w- injective module.

Corollary 2.2 :- The following conditions are equivalent for ring R :

1. R is right or left pseudo w- injective
2. If r(x)=r(y) with x, y ∈ R then Rx=Ry

Then the relation is as follows :

Quasi w- injective →quasi pseudo w- injective →quasi pseudo GP-w- injective

Example 2.3 :-

1) Let F be an algebraically closed field and a, b be indeterminates. Let R= F(b)[a] such that af – fa = df/dy , f ∈ F(b) then the R-mod M = $\frac{R}{(a(a+b)(a+b-1)/b)}$ R is quasi pseudo w-injective but not quasi w-injective.
2) Let K=F(z_1, z_2…………..) and L=F(z_2, z_3……………) with F is a field and ρ : K→L be an isomorphism ρ (bi) = bi+1 and ρ (c) = c for all c ∈ F. Let K[a_1,a_2 ; ρ] be the ring of twisted left polynomials over K where aiK = ρ (k)ai for all k ∈ K and I = 1,2. Set R = K[a1,a2 ; ρ] /(a_1^2, a_2^2). Then R_R is quasi pseudo GP-w- injective which is not quasi pseudo w- injective

Now we study some properties of quasi pseudo GP-w- injective Self generator modules and their Endomorphism rings.

Theorem 2.4 :- Let M be a right R- Module with S= End (M) then

a) If S is a right or left pseudo GP-w injective ring then M is quasi pseudo GP-w injective.
b) If M is quasi pseudo GP-w injective and self generator then S is a right or left pseudo GP-w injective ring.

Proof :- (a) Let f ∈ S since S is right or left pseudo GP-w injective there exists n∈ N such that $f^n \neq 0$ and if $rs(f^n) = r_s(g)$ for some g∈S then g∈ Sfn by corollary 1.2. Assume that Kerfn=Kerg with g∈S then $r_s(f^n)=r_s(g)$ and hence $g \in Sf^n$ thus M is quasi pseudo GP-w injective by corollary 1.2. (b) Let 0≠ f ∈ S since M is quasi pseudo GP-w injective there exist n∈N such that fn≠0 and if $Ker(f^n)$ = Ker (g) by our assumption $g \in f^n$ and so S is right pseudo GP-w injective.

Corollary 2.5 :- Let M be a right R module with S= End(M) then

1) If S is a right pseudo w-injective ring then M is quasi pseudo w- injective
2) If M is a quasi pseudo w-injective module which is a self generator then S is a right pseudo w-injective

For a right R module M,S=End (M) W(S) = { s ∈ S │Ker (s) is essential in M}

Lemma 2.6 :- Let M_R be a quasi pseudo GP-w injective module which is a self generator S= End (M). if a ∉W(s) then Ker(a) < Ker(a-ata) for some t ∈ S.

Lemma 2.7 :- Assume that M is quasi pseudo GP-w injective module which is a self generator then J(s) =W (s).

Corollary :- if is right or left pseudo GP-w injective then J(R) =$Z(R_R)$.

Theorem 2.7:- Let M be a quasi pseudo GP w-injective module which is a self generator S =End (M) then the following conditions are equivalent :

1. S is right perfect then it is strongly perfect.
2. For any infinite sequence k_1,k_2……. ∈ S the chain Ker(k1)≤ Ker(k2) ≤ …….. is stationary.

Proposition :- Let M be a quasi pseudo GP-w injective self generator module and S= End (M) if M satisfies ACC on M- annihilators then S is semi primary.

Conclusion :- On the study of quasi pseudo injective module we can generalize it for quasi pseudo w injective module and then generalized principally for w injective module.

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Reference

1. F.W. Anderson, K.R.Fuller, " Rings and Categories of Modules", Springer-Verlag. New York 1974
2. J. Chen, N. Ding, On general principally injective rings, Comm. Algebra 1999
3. J. Chen, Y.Q. Zhou and Zhu, GP-injective rings need not be w-injective rings, Comm. Algebra 33
4. K. Kim, S.B. Nam and J.Y.Kim, On simple singular GP-injective Modules, Comm. Algebra.
5. S.K. Jain, S. Singh, quasi injective and pseudo injective modules, Canad. Math. Bull. 18
6. W.K. Nicholson, J.K. Park and M.F. Yousif, Principally quasi injective Modules, Comm. Algebra 27 (1) 1999
7. B. Samruam, H.D. Hai, N.V.Sanh, A general form of pseudo p- injectivity and w- injectivity. Southest Asian Bull. Of Math, 35 (2011)
8. N.V.Sanh and K.P. Shum,S. Dhompongsa and S. Wongwai, On quasi principally injective modules and rings, Algebra Colloquium (1999)
9. S.Singh, S.K.Jain, On pseudo injective modules and self pseudo injective rings, The J. of Math. Sciences.

10. H.Q.Dinh, N. Ding A note on pseudo injective Modules with annihilator conditions , Comm. Algebra (2005).