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RESULTS ON HESITANT FUZZY SOFT TOPOLOGICAL SPACES

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ABSTRACT

In this paper, we define Hesitant Fuzzy Soft (HF Soft shortly) Topological Space, HF Soft T1- Space, HF Soft Hausdorff space, HF Soft normal space, HF Soft regular space and we study countable HF Soft basis, First and second countable HF Soft spaces. We also give the definitions for Separation axioms, HF Soft Connected space, HF Soft pu continuous mapping, HF Soft pu open and closed mapping and study some significant properties related to them.

KeyWords
HF Soft Topological space, First countable HF soft space, HF soft pu mapping, HF soft T_0 space, HF soft T_1space, HF soft Hausdorff space, HF Soft normal space, HF Soft basis.

1. INTRODUCTION

Imperfection, uncertainty and ambiguity are essential factors in the real life. The significance and benefits of uncertainty is noticed by number of researchers for many years. It is identified as a complicated one to be handled within some constraints. To cope up with uncertainty, various approaches including fuzzy, rough and vague theory are introduced and improved in different points of view. [2] Molodtsov's soft set theory is also one kind of these approaches. It is established with a new frame work to deal ambiguity. It mainly concentrates on parameterization. Without any particular condition on parameters, the soft theory has developed in representing uncertain concepts appropriately. In 1965, a new mathematical tool to deal with uncertainty, [1] Zadeh has introduced fuzzy sets and this concept is used in many fields. A generalization of fuzzy sets is proposed by Torra [6]. They are hesitant fuzzy sets which allow the membership degree to have a set of possible values and the membership is a subset of the closed interval [0, 1]. This new topic has been magnetized the attention of numerous authors. [7] Verma and Sharma have defined set theoretic operations on hesitant fuzzy sets. J. Wang et al. has also defined few operations on Hesitant fuzzy soft sets to use them in multi criteria group decision making problems. In 2013 K. V. Babithaa and Sunil Jacob John [14] introduced the combination of hesitant fuzzy sets and soft sets, i.e. the notion of hesitant fuzzy soft sets. Hesitant fuzzy topological space and some related properties are introduced by Serkan Karata [15].

In this paper, we define HF Soft T0- Space, HF Soft T1- Space, HF Soft T2- Space or HF Soft Hausdorff space, HF Soft T3- Space, HF Soft regular space, HF Soft normal space and HF Soft T4- Space and also study countable HF Soft basis, first countable HF Soft space and second countable HF Soft space and few of their properties. Finally we give the HF Soft separation axioms, then HF Soft connected space, HF Soft pu continuous mapping, HF Soft pu open mapping and HF Soft pu closed mapping and study various properties.

Let us suppose that X is any universal set and E is a set of parameters specified by the elements of X throughout the paper.

2. Preliminaries

Definition 2.1. [2] A soft set is defined as a pair (s, A), denoted by sX^A over X, where s: A $\to \rho$(X), A $\subseteq$ E, ρ(X) is the set of all subsets of X. The class of all soft sets defined on X with respect to the parameters in A is denoted by $s\tilde{X}^A$.

Definition 2.2. [16] A fuzzy soft set is defined as a pair ($\widetilde{fs}$, A) denoted by $\widetilde{fs}X^A$ over X, where $\widetilde{fs}$: A $\to$ ρ(X), A ⊆ E, ρ(X) is a set of all fuzzy sets in X. The class of all fuzzy soft sets over X with respect to the parameters in A is denoted by $\widetilde{fs}(\tilde{X}^A)$.

Definition 2.3 [6] A hesitant fuzzy set is defined as a function $\widetilde{hf}$ that when applied on X returns a subset of [0, 1], i.e. $\widetilde{hf}$: X → [0, 1]. A hesitant fuzzy set over X is denoted by $\widetilde{hf}^X$ and the class of all hesitant fuzzy sets defined over X is denoted by $\widetilde{hf}(\tilde{X})$.

Definition 2.4. [6] The lower and upper bound of a hesitant fuzzy set $\widetilde{hf}^X$ over X are defined as below.

Lower bound of $\widetilde{hf}^X$ is defined and denoted by $\widetilde{hf}^{X-}(\square)$ = min {$\widetilde{hf}(\square)$: for all $x \in X$}.

Upper bound of $\widetilde{hf}^X$ is defined and denoted by $\widetilde{hf}^{X+}(\square)$ = max {$\widetilde{hf}(\square)$: for all $x \in X$}.

Definition 2.5. [6] The complement of any hesitant fuzzy set $\widetilde{hf}^X$(x) is defined as $\widetilde{hf}^X(\square) = \bigcup_{\gamma \in \widetilde{hf}x}\{1 - \gamma\}$.

Definition 2.6. [6] The union of two hesitant fuzzy sets $\widetilde{hf}^{X1}$ and $\widetilde{hf}^{X2}$ is a hesitant fuzzy set $\widetilde{hf}^{X1} \cup \widetilde{hf}^{X2}$ such that $(\widetilde{hf}^{X1} \cup \widetilde{hf}^{X2})(x)$ = {h ∈($\widetilde{hf}^{X1}$(x) ∪ $\widetilde{hf}^{X2}$(x)) / h ≥ max($\widetilde{hf}^{X1-}$, $\widetilde{hf}^{X2-}$)}.

Definition 2.7. [6] Let $\widetilde{hf}^{X1}$, $\widetilde{hf}^{X2}$ ∈ $\widetilde{hf}(\tilde{X})$. Then $\widetilde{hf}^{X1}$ is said to be a subset of $\widetilde{hf}^{X2}$ if $\widetilde{hf}^{X1}$(x) ⊆ $\widetilde{hf}^{X2}$(x) for all x ∈X and is denoted by $\widetilde{hf}^{X1} \subseteq \widetilde{hf}^{X2}$.

Definition 2.8. [6] Two hesitant fuzzy sets $\widetilde{hf}^{X1}$, $\widetilde{hf}^{X2}$ are said to be equal if $\widetilde{hf}^{X1} = \widetilde{hf}^{X2}$ if and only if $\widetilde{hf}^{X1} \subseteq \widetilde{hf}^{X2}$ and $\widetilde{hf}^{X2} \subseteq \widetilde{hf}^{X1}$.

Definition 2.9. [6] The intersection of two hesitant fuzzy sets $\widetilde{hf}^{X1}$ and $\widetilde{hf}^{X2}$ is a hesitant fuzzy set $\widetilde{hf}^{X1} \cap \widetilde{hf}^{X2}$ such that $(\widetilde{hf}^{X1} \cap \widetilde{hf}^{X2})(\square)$ = {h ∈($\frac{\widetilde{hf}X1(x) \cup \widetilde{hf}X2(x)}{h}$) / h ≤ min($\widetilde{hf}^{X1+}$, $\widetilde{hf}^{X2+}$)}.

Definition 2.10. [6] We define the hesitant fuzzy null set as a HF set such that $\widetilde{hf}$(x) = {0} for all x ∈X. It is denoted by $\widetilde{hf}^{\varphi}$.

Definition 2.11. [6] We define the hesitant fuzzy full set as a HF set such that $\widetilde{hf}$(x) = {1} for all x ∈X. It is denoted by $\widetilde{hf}^{X}$.

Definition 2.12. [14] A pair ($\widetilde{hfs}$, A) is called a hesitant fuzzy soft set (HF soft set in short cut) if $\widetilde{hfs}$ is a mapping such that $\widetilde{hfs}$: A → $\widetilde{hf}(\tilde{X})$. i.e. $\widetilde{hfs}$(e) ∈ $\widetilde{hf}(\tilde{X})$, for all e ∈ A where A ⊆ E. The set of all hesitant fuzzy soft sets is denoted by $\widetilde{hfs}(\tilde{X}^E)$.

Example 2.13. let X be set of team players played in a tournament. X ={p1, p2, p3, p4}. Let A = {consistency, coordination, all rounder}.

Then hesitant fuzzy soft set $\widetilde{hfs}X_A$ given below is the assessment of the performance of 4 team members p1, p2, p3, p4 given by three referees.

$\widetilde{hfs}$(Consistency) = {p1 = {0.6,0.9,0.8}, p2 = {0.6,0.5,0.8}, p3 = {0.9,0.7,0.9}, p4 = {0.5,0.9,0.8}} $\widetilde{hfs}$(coordination) = {p1 = {0.5,0.7,0.5}, p2 = {0.6,0.7,0.75}, p3 = {0.85,0.9,0.8}, p4 = {1,0.9,0.9}} $\widetilde{hfs}$(all rounder) = {p1 = {0.6,0.7,0.9}, p2 = {0.6,0.7,0.8}, p3 = {0. 8,0.9,0.8}, p4 = {0.9,0.65,0.9}}.

Definition 2.14. [14] For two hesitant fuzzy soft sets $\widetilde{hfs}X^A$ and $\widetilde{hfq}X^B$ over a common universe X, we say that $\widetilde{hfs}X^A$ is hesitant fuzzy soft subset of $\widetilde{hfq}X^B$. It is denoted by $\widetilde{hfs}X^A \subseteq \widetilde{hfq}X^B$.

if (i) ⍰ ⊆ ⍰ (ii) $\widetilde{hfs}$ (⍰) is hesitant fuzzy sub set of $\widetilde{hfq}$ (⍰) for every a in A.

Example 2.15. Let X is set of team members involved in a project. X ={p1, p2, p3, p4}. Let B = {consistency, coordination, leadership, all rounder}.

Then hesitant fuzzy soft set $\widetilde{hfq}X^B$ given below is the assessment of the performance of 4 team members p1, p2, p3, p4 given by three referees.

$\widetilde{hfq}$(consistency) = {p1 = (0.85,1,0.8), p2 = (0.75,0.65,0.7), p3 = (0.8,0.75,0.99), p4 = (0.8,0.9,0.85)}; $\widetilde{hfq}$(coordination) = {p1 = (0.75,0.75,0.6), p2 = (0.85,0.8,0.8), p3 = (0.9,0.9,0.8), p4 = (1,0.9,0.99)}; $\widetilde{hfq}$(leadership) = {p1 = (0.85,0.8,0.9), p2 = (0.8,0.8,0.9), p3 = (0. 8,0.9,0.95), p4 = (0.9,0.75,0.85)}; $\widetilde{hfq}$(all rounder) = {p1 = (0.85,0.9,0.9), p2 = (0.65,0.7,0.8), p3 = (0. 85,0.9,0.9), p4 = (0.9,0.75,0.8)}.

From Example 2.13 and Example 2.15 we can observe that $\widetilde{hfs}X^A \subseteq \widetilde{hfq}X^B$.

Definition 2.16. [14] Let $\widetilde{hfs}X^A$ be hesitant fuzzy soft set. Then the complement of $\widetilde{hfs}X^A$ is defined by $\widetilde{hfs}X^{Ac}$ = ($\widetilde{hf}^{⍰}$, ⍰) where $\widetilde{hf}^c$(⍰) is the complement of the hesitant fuzzy set $\widetilde{hf}$(⍰).

Note. (i). Let $\widetilde{hfs}X^A$ and $\widetilde{hfq}X^B$ be two hesitant fuzzy soft sets over X. Then intersection, union, AND or OR of $\widetilde{hfs}X^A$ and $\widetilde{hfq}X^B$ are hesitant fuzzy soft sets and they satisfy Demorgan laws.

(ii). $[\widetilde{hfs}X^A \cup \widetilde{hfq}X^B]^c = (\widetilde{hfs}X^A)^c \cap (\widetilde{hfq}X^B)^c$.

(iii). $[\widetilde{hfs}X^A \cap \widetilde{hfq}X^B]^c = (\widetilde{hfs}X^A)^c \cup (\widetilde{hfq}X^B)^c$.

3. HESITANT FUZZY SOFT FIRST COUNTABLE SPACE

Definition 3.1. The hesitant fuzzy soft null set is a HF soft set $\widetilde{hfs}$ X^A over X is said to be HF soft null set denoted by $\widetilde{hfs}X^{\varphi}$ if ∀e ∈ A, $\widetilde{hfs}$ (e) is the hesitant fuzzy null set $\widetilde{hf}^{\varphi}$ of X where $\widetilde{hf}$ (x) = {0}, ∀ x ∈ X.
Definition 3.2. A HF soft set $\widetilde{hfs}$ X^A over X is said to be HF soft full set denoted by $\widetilde{hfs}$ X^X if ∀e ∈ A, $\widetilde{hfs}$ (e) is the hesitant fuzzy full set $\widetilde{hf}^{X}$ of X where $\widetilde{hf}$ (x) = {1}, ∀ x ∈ X.

In this section, we will introduce hesitant fuzzy soft topological spaces and related properties.

Definition 3.3.[15] A hesitant fuzzy topology HFτ is a family of hesitant fuzzy sets over X which satisfies the following conditions:

(i) $\widetilde{hf}^{\varphi}$, $\widetilde{hf}^{X}$ ∈ HFτ,

(ii) If $\widetilde{hf}^{A}$, $\widetilde{hf}^{B}$ ∈ HFτ , then $\widetilde{hf}^{A}$ ∩ $\widetilde{hf}^{B}$ ∈ HFτ,

(iii) If { $\widetilde{hf}^{Ai}$}$_{i∈I}$ ⊆ HFτ , then $\bigcup_{i∈I}$ $\widetilde{hf}^{Ai}$ ∈ HFτ .

The pair (HF$\widetilde{XE}$, HFτ) is called a hesitant fuzzy topological space. Every element of HFτ is called hesitant fuzzy open set. If the complement of a hesitant fuzzy set is a hesitant fuzzy open set, then the hesitant fuzzy set is called hesitant fuzzy closed set.
Definition 3.4. A hesitant fuzzy soft topology HFSτ is a family of hesitant fuzzy soft sets over X which satisfies the following conditions:

(i) $\widetilde{hfs}X^{\varphi}$, $\widetilde{hfs}$ X^X ∈ HFSτ,

(ii) If $\widetilde{hfs}$ X^A, $\widetilde{hfs}$ X^B ∈ HFSτ, then $\widetilde{hfs}$ X^A ∩ $\widetilde{hfs}$ X^B ∈ HFSτ,

(iii) If {$\widetilde{hfs}$ X^{Hi} }$_{i∈I}$ ⊆ HFSτ, then $\bigcup_{i∈I}$ $\widetilde{hfs}$ X^{Hi} ∈ HFSτ .

The pair (HFSX_E, HFSτ) is called a hesitant fuzzy soft topological space. Every element of HFSτ is called hesitant fuzzy soft open set.

Definition 3.5. If the complement of a HF soft set $\widetilde{hfs}$ X^A is a hesitant fuzzy soft open set, then $\widetilde{hfs}$ X^A is called HF soft closed set.
Definition 3.6. A sequence of HF Soft sets, {($\widetilde{hfs}X^{Gn}$, A) : n ∈ N} is eventually contained in a HF Soft set $\widetilde{hfs}X^A$ if and only if there is an integer m such that ($\widetilde{hfs}X^{Gn}$, A) ⊆ $\widetilde{hfs}X^A$ for all n ≥ m. The sequence is frequently contained in $\widetilde{hfs}X^A$ if and only if for each integer m, there is an integer n such that n ≥ m and ($\widetilde{hfs}X^{Gn}$, A) ⊆ $\widetilde{hfs}X^A$. If the sequence is in a HF Soft topological space (HFS$\widetilde{XE}$, HFSτ), then we say that the sequence converges to a soft set $\widetilde{hfs}X^A$ if it is eventually contained in each neighbourhood of $\widetilde{hfs}X^A$.

Definition 3.7. Let g be a mapping over the set of non-negative integers. Then the sequence {($\widetilde{hfq}X^{Gi}$, A): i ∈ N} of HF Soft sets is a subsequence of a sequence {($\widetilde{hfs}X^{Gn}$, A) : n ∈ N} of HF Soft sets if and only if there is a map g such that ($\widetilde{hfq}X^{Gn}$, A) = ($\widetilde{hfs}X^{Gg(m)}$, A) and for each integer m, there is an integer K such that g(m) ≥ n whenever n ≥ K.
Definition 3.8. A HF Soft set $\widetilde{hfs}X^A$ in a HF Soft topological space (HFS$\widetilde{XE}$, HFSτ) is a cluster HF Soft set of a sequence of HF Soft sets if the sequence is frequently contained in each neighbourhood of $\widetilde{hfs}X^A$.
Definition 3.9. A HF Soft topological space (HFS$\widetilde{XE}$, HFSτ) is called a HF Soft T_0-space if for every distinct points $\widetilde{hfs}X^A$e1, $\widetilde{hfs}X^A$e2 ∈HFS$\widetilde{XE}$ and for every a ∈A there exists two HF Soft open sets $\widetilde{hfq}X^{B1}$ and $\widetilde{hfq}X^{B2}$ such that $\widetilde{hfs}X^A$e1 ∈$\widetilde{hfq}X^{B1}$, $\widetilde{hfs}X^A$e2 ∉ $\widetilde{hfq}X^{B1}$ or
$\widetilde{hfs}X^A$e2 ∈$\widetilde{hfq}X^{B2}$ and $\widetilde{hfs}X^A$e1 ∉ $\widetilde{hfq}X^{B}$.

Definition 3.10. A HF Soft topological space (HFS$\widetilde{XE}$, HFSτ) is called a HF Soft T_1-space if for every distinct points $\widetilde{hfs}X^Ae1$, $\widetilde{hfs}X^Ae2$ ∈HFS$\widetilde{XE}$ and for every a ∈A there exists two HF Soft open sets $\widetilde{hfq}X^{B1}$ and $\widetilde{hfq}X^{B2}$ such that $\widetilde{hfs}X^Ae1 \in \widetilde{hfq}X^{B1}$, $\widetilde{hfs}X^Ae2 \notin \widetilde{hfq}X^{B1}$ and
$\widetilde{hfs}X^Ae2 \in \widetilde{hfq}X^{B2}$ and $\widetilde{hfs}X^Ae1 \notin \widetilde{hfq}X^{B2}$.

Definition 3.11. A HF Soft topological space (HFS$\widetilde{XE}$, HFSτ) is called a HF Soft T_2-space or Hausdorff space if for every distinct points $\widetilde{hfs}X^Ae1$, $\widetilde{hfs}X^Ae2$ ∈HFS$\widetilde{XE}$ and for every a ∈A there exists two HF Soft open sets $\widetilde{hfq}X^{B1}$ and $\widetilde{hfq}X^{B2}$ such that $\widetilde{hfs}X^Ae1 \in \widetilde{hfq}X^{B1}$ and $\widetilde{hfs}X^Ae2 \in \widetilde{hfq}X^{B2}$ and $\widetilde{hfq}X^{B1} \cap \widetilde{hfq}X^{B2} = \widetilde{hfsX^{\varphi}}$.

Definition 3.12. A HF Soft topological space (HFS$\widetilde{XE}$, HFSτ) is called a HF Soft regular-space over X if there is a HF Soft closed set $\widetilde{hfq}X^B$ in HFS$\widetilde{XE}$ such that $\widetilde{hfs}X^Ae1$,∈ HFS$\widetilde{XE}$ and $\widetilde{hfs}X^Ae1 \notin \widetilde{hfq}X^B$. For every a ∈ A if there exists two HF Soft open sets $\widetilde{hfq}X^{B1}$ and $\widetilde{hfq}X^{B2}$ such that $\widetilde{hfs}X^Ae1 \in \widetilde{hfq}X^{B1}$, $\widetilde{hfq}X^B \subseteq \widetilde{hfq}X^{B2}$ and $\widetilde{hfq}X^{B1} \cap \widetilde{hfq}X^{B2} = \widetilde{hfsX^{\varphi}}$.

Definition 3.13. A HF Soft topological space (HFS$\widetilde{XE}$, HFSτ) is said to be a HF Soft T_3-space over X if it is HF Soft regular and HF Soft T1- space.

Definition 3.14. Let (HFS$\widetilde{XE}$, HFSτ) be a HF Soft topological space over X, $\widetilde{hfq}X^{B1}$ and $HFQX^{B2}$ be HF Soft closed sets over X such that $\widetilde{hfq}X^{B1} \cap \widetilde{hfq}X^{B2} = \widetilde{hfsX^{\varphi}}$.If there exists HF Soft open sets $\widetilde{hfs}X^{A1}$ and $\widetilde{hfs}X^{A2}$ such that $\widetilde{hfq}X^{B1} \subseteq \widetilde{hfs}X^{A1}$, $\widetilde{hfq}X^{B2} \subseteq \widetilde{hfs}X^{A2}$ and
$\widetilde{hfs}X^{A1} \cap \widetilde{hfs}X^{A2} = \widetilde{hfsX^{\varphi}}$, then (HFS$\widetilde{XE}$, HFSτ) is said to be HF Soft normal space.

Definition 3.15. If a HF Soft topological space is both HF Soft T1- Space and HF Soft normal space then it is called a HF Soft T4- Space.

Definition 3.16. Let (HFS$\widetilde{XE}$, HFSτ) be a HF Soft topological space and $\widetilde{hfs}X^A$ ∈HFS$\widetilde{XE}$. Then a countable HF Soft basis at the HF Soft set, $\widetilde{hfs}X^A$ is a sequence {($\widetilde{hfs}X^{Gn}$, A) : n ∈ N} of neighborhoods of $\widetilde{hfs}X^A$ such that if $HFNX^A$ is a neighborhood of $\widetilde{hfs}X^A$, then $\widetilde{hfs}X^{Gn} \subseteq \widetilde{hfn}X^A$ for some n. The sets $\widetilde{hfs}X^{Gn}$ need not be distinct.

Definition 3.17. A space (HFS$\widetilde{XE}$, HFSτ) is first-countable HF Soft space, if it has a countable HF Soft basis at each point x ∈X.

Definition 3.18. A space (HFS$\widetilde{XE}$, HFSτ) is second-countable HF Soft space, if it has a countable HF Soft basis for its HF Soft topology, say {($\widetilde{hfs}X^{Gn}$, A) : n ∈ N},That is, given any HF Soft open set $\widetilde{hfs}X^A$ and HF Soft point $\widetilde{hfs}X^Ae \in \widetilde{hfs}X^A$, there is $\widetilde{hfs}X^{Gn} \subseteq \widetilde{hfs}X_A$ with $\widetilde{hfs}X^Ae \in \widetilde{hfs}X^{Gn}$.

Theorem 3.19. If the neighbourhood system of each HF Soft set in a HF Soft topological space (HFS$\widetilde{XE}$, HFSτ) is countable, then
(a) A HF Soft set $\widetilde{hfs}X^A$ is HF Soft open if and only if each sequence {($\widetilde{hfs}X^{Gn}$, A): n = 1, 2,….} of HF Soft sets which converges to a HF Soft set $\widetilde{hfq}X^A$ contained in $\widetilde{hfs}X^A$ is eventually contained in $\widetilde{hfs}X^A$.
(b) If $\widetilde{hfs}X^A$ is a cluster HF Soft set of a sequence {($\widetilde{hfs}X^{Gn}$, A): n = 1, 2, …} of HF Soft sets, then there is a subsequence of the sequence converging to $\widetilde{hfs}X^A$.

Proof. (a) ⇒Since $\widetilde{hfs}X^A$ is open, $\widetilde{hfs}X^A$ is a neighbourhood of $\widetilde{hfq}X^A$. Hence, {($\widetilde{hfs}X^{Gn}$, A) : n = 1, 2, …} is eventually contained in $\widetilde{hfs}X^A$.
Conversely For each $\widetilde{hfq}X^A \subseteq \widetilde{hfs}X^A$, let $\widetilde{hfq1}X^A$, $\widetilde{hfq2}X^A$,…., $\widetilde{hfqn}X^A$, …. be the HF Soft neighbourhood system $\widetilde{hfq}X^A$.
Let $\widetilde{hfm_n}X^A$ = ∩(Gi,A), I = 1, 2,…n,.
Then $\widetilde{hfm_1}X^A$, $\widetilde{hfm_2}X^A$, ….,$\widetilde{hfm_n}X^A$,……
is a sequence which is eventually contained in each neighbourhood of $\widetilde{hfq}X^A$.
i.e., $\widetilde{hfm_1}X^A$, $\widetilde{hfm_2}X^A$, ….,$\widetilde{hfm_n}X^A$,…… converges to $\widetilde{hfq}X^A$.
Hence, there is an integer m such that for n ≥ m, $\widetilde{hfm_n}X^A \subseteq \widetilde{hfs}X^A$.
Then $\widetilde{hfm_n}X^A$'s are neighbourhoods of $\widetilde{hfq}X^A$.
Therefore $\widetilde{hfs}X^A$ is HF Soft open.
(b) Let $\widetilde{hfk_1}X^A$, $\widetilde{hfk_2}X^A$, …,$\widetilde{hfk_n}X^A$,…… be the neighbourhood system of $\widetilde{hfs}X^A$ and let $\widetilde{hfl_n}X^A = \cap\ \widetilde{hfk_i}X^A$ I = 1,2,…n…
Then $\widetilde{hfl_1}X^A$, $\widetilde{hfl_2}X^A$,…., $\widetilde{hfl_n}X^A$,…. is a sequence such that $\widetilde{hfl_{n+1}}X^A \subseteq \widetilde{hfl_n}X^A$ for all n.
For each non-negative integer i, choose g(i) such that g(i) ≥ i and ($\widetilde{hfs}X^{Ag(i)}$ A) ⊆ $\widetilde{hfl_i}X^A$.
Then surely {($\widetilde{hfs}X^{f(i)}$ A): i = 1, 2,….} is a subsequence of the sequence {($\widetilde{hfl_i}X^A$,A) : n = 1, 2,….}.
And clearly this subsequence converges to $\widetilde{hfs}X^A$.

4. HFS SEPARATION AXIOMS AND CONNECTED SPACE

This is a natural phenomenon that a topologist wishes to prove or disprove some significant results of general topology of the soft set form, fuzzy soft set form and hesitant fuzzy soft set form. Here we introduce and exhibit some concepts of the hesitant fuzzy soft connected spaces and related results.

Definition 4.1. Let (HFS$\widetilde{XE}$, HFSτ) be a HF Soft topological space over X. A HF Soft separation of HFS$\widetilde{XE}$is a pair $\widetilde{hfs}X^{E1}$ and $\widetilde{hfs}X^{E2}$ of no-null HF Soft open sets such
That HFS$\widetilde{XE}$= $\widetilde{hfs}X^{E1} \cup \widetilde{hfs}X^{E2}$ and $\widetilde{hfs}X^{E1} \cap \widetilde{hfs}X^{E2} = \widetilde{hfsX^{\varphi}}$.

Defnition 4.2. A HF Soft topological space (HFS$\widetilde{XE}$, HFSτ) is said to be HF Soft connected if there does not exist a HF Soft separation of HFS$\widetilde{XE}$. Otherwise, (HFS$\widetilde{XE}$, HFSτ) is said to be HF Soft disconnected.

Theorem 4.3. HF Soft topological space (HFS$\widetilde{XE}$, HFSτ) is HF Soft connected if and only if the only HF Soft sets in HFS$\widetilde{XE}$ that are both HF Soft open and HF Soft closed are $\widetilde{hfsX^{\varphi}}$ and $\widetilde{hfsX^{X}}$.

Proof. Let (HFS$\widetilde{XE}$, HFSτ) be HF Soft connected. Suppose in a contrary way that $\widetilde{hfs}X^A$ is a HF Soft set which is both HF Soft open and HF Soft closed in HFS$\widetilde{XE}$ other than $\widetilde{hfsX^{\varphi}}$ and $\widetilde{hfsX^{X}}$. Clearly, $(\widetilde{hfs}X^A)^c$ is a HF Soft open set in HFS$\widetilde{XE}$ different from $\widetilde{hfsX^{\varphi}}$ and $\widetilde{hfsX^{X}}$. Also we know that $\widetilde{hfs}X^A \cup (\widetilde{hfs}X^A)^c = \widetilde{hfsX^{X}}$ and $\widetilde{hfs}X^A \cap (\widetilde{hfs}X^A)^c = \widetilde{hfsX^{\varphi}}$.

Therefore we have $\widetilde{hfs}X^A$, $(\widetilde{hfs}X^A)^c$ is a HF Soft separation of HFS$\widetilde{XE}$.

This is a contradiction. Thus the only HF Soft closed and open sets in HFS$\widetilde{XE}$ are $\widetilde{hfsX^{\varphi}}$ and $\widetilde{hfsX^{X}}$.

Conversely, let $\widetilde{hfs}X^{A1}$, $\widetilde{hfs}X^{A2}$ be a HF Soft separation of HFS$\widetilde{XE}$.

$\Rightarrow \widetilde{hfsX^{X}} = \widetilde{hfs}X^{A1} \cup \widetilde{hfs}X^{A2}$ and $\widetilde{hfs}X^{A1} \cap \widetilde{hfs}X^{A2} = \widetilde{hfsX^{\varphi}}$.

Let $\widetilde{hfs}X^{A1} = \widetilde{hfsX^{X}}$. Then $\widetilde{hfs}X^{A2} = \widetilde{hfsX^{\varphi}}$.

This is a contradiction. Hence, $\widetilde{hfs}X^{A1} \neq \widetilde{hfsX^{X}}$.

Therefore $\widetilde{hfs}X^{A1} = (\widetilde{hfs}X^{A2})^c$.

This shows that $\widetilde{hfs}X^{A1}$ is both HF Soft open and HF Soft closed other than the null and full HF Soft sets $\widetilde{hfsX^{\varphi}}$ and $\widetilde{hfsX^{X}}$.

This is a contradiction.

Therefore (HFS$\widetilde{XE}$, HFSτ) is HF Soft connected.

Example 4.4. Since the only HF Soft sets in HFS$\widetilde{XE}$ that are both HF Soft open and HF Soft closed are $\widetilde{hfsX^{\varphi}}$ and $\widetilde{hfsX^{X}}$, HF Soft indiscrete topological space (HFS$\widetilde{XE}$, HFSτ) is HF Soft connected.

Example 4.5. HF Soft discrete topological space (HFS$\widetilde{XE}$, HFSτ) is HF Soft disconnected. Because for at least one HF Soft set $\widetilde{hfs}X^A$ in HFS$\widetilde{XE}$ is both HF Soft open set and HF Soft closed.

Corollary 4.6. Let (HFS$\widetilde{XE}$, HFSτ) be a HF Soft topological space over X. Then following statements are equivalent.

1. (HFS$\widetilde{XE}$, HFSτ) is HF Soft connected.
2. No-null HF Soft open sets $\widetilde{hfs}X^{A1}$, $\widetilde{hfs}X^{A2}$ and $\widetilde{hfsX^{X}} = \widetilde{hfs}X^{A1} \cup \widetilde{hfs}X^{A2}$ but $\widetilde{hfs}X^{A1} \cap \widetilde{hfs}X^{A2} \neq \widetilde{hfsX^{\varphi}}$.
3. The only HF Soft sets in HFS$\widetilde{XE}$ that are both HF Soft open and HF Soft closed in HFS$\widetilde{XE}$ are $\widetilde{hfsX^{\varphi}}$ and $\widetilde{hfsX^{X}}$.
4. $\widetilde{hfsX^{X}} = \widetilde{hfs}X^{A1} \cup \widetilde{hfs}X^{A2}$ but $\widetilde{hfs}X^{A1} \cap \widetilde{hfs}X^{A2} = \widetilde{hfsX^{\varphi}}$ then $\widetilde{hfs}X^{A1} = \widetilde{hfsX^{\varphi}}$ or $\widetilde{hfs}X^{A2} = \widetilde{hfsX^{\varphi}}$.
5. $\widetilde{hfsX^{X}} = \widetilde{hfs}X^{A1} \cup \widetilde{hfs}X^{A2}$ but $\widetilde{hfs}X^{A1} \cap \widetilde{hfs}X^{A2} = \widetilde{hfsX^{\varphi}}$ then $\widetilde{hfs}X^{A1} = \widetilde{hfsX^{X}}$ or $\widetilde{hfs}X^{A2} = \widetilde{hfsX^{X}}$.

5. HFS PU-CONTINUOUS FUNCTIONS

We introduce the notion of HF Soft pu-continuity of functions that are induced by two mappings u: $\widetilde{hfs}X^A \rightarrow \widetilde{hfq}Y^B$ and p: A → B on HF Soft topological spaces (HFS$\widetilde{XE}$, HFSτ) and (HFQ$\widetilde{YF}$, HFSτ*).

Definition 5.1. Let (HFS$\widetilde{XE}$, HFSτ) and (HFQ$\widetilde{YF}$, HFSτ*) be HF Soft topological spaces. Let u: $\widetilde{hfs}X^A \rightarrow \widetilde{hfq}Y^B$ and p: A → B be two mappings. Let f^{pu}: HFS$\widetilde{XE}$ → HFQ$\widetilde{YF}$ be a HF soft function and $\widetilde{hfs}X^Ae \in \widetilde{hfs}X^A$.

(i) f^{pu} is HF Soft pu-continuous at a HF Soft point $\widetilde{hfs}X^Ae \in \widetilde{hfs}X^A$ if for each $\widetilde{hfq}X^B \in N_{HFS\tau^*}(f^{pu}(\widetilde{hfs}X^Ae))$, there exists a $\widetilde{hfm}X^A \in N_{HFS\tau}(\widetilde{hfs}X^Ae)$ such that $f^{pu}(\widetilde{hfm}X^A) \subseteq \widetilde{hfq}X^B$.

(ii) f^{pu} is HF Soft pu-continuous on $\widetilde{hfs}X^A$ if f^{pu} is HF Soft continuous at each HF Soft point in $\widetilde{hfs}X^A$.

(iv) f^{pu} is HF Soft pu-open mapping if for each HF Soft open set $\widetilde{hfs}X^A \in$ HFS$\widetilde{XE}$, if

$f^{pu}(\widetilde{hfs}X^A)$ is HF Soft open set in HFQ$\widetilde{YF}$.

Theorem 5.2. Let (HFS$\widetilde{XE}$, HFSτ) and (HFQ$\widetilde{YF}$, HFSτ*) be HF Soft topological spaces. Let f^{pu}: HFS$\widetilde{XE}$ → HFQ$\widetilde{YF}$ be a function and $\widetilde{hfs}X^Ae \,\widetilde{\in}\, \widetilde{hfs}X^A$. Then the following statements are equivalent.

(i) f^{pu} is HF Soft pu-continuous at $\widetilde{hfs}X^Ae$;

(ii) For each $\widetilde{hfq}X^B \in N_{HFS\tau^*}(f_{pu}(\widetilde{hfs}X^Ae))$, there exists a $\widetilde{hfm}X^A \in N_{HFS\tau}(\widetilde{hfs}X^Ae)$ such that $\widetilde{hfm}X^A \subseteq (f^{pu})^{-1}(\widetilde{hfq}X^B)$.

(iii) For each $\widetilde{hfqX^B} \in N_{HFS\tau^*}(f^{pu}(\widetilde{hfs}X^Ae))$, $(f^{pu})^{-1}(\widetilde{hfq}X^B) \in N_{HFS\tau}(\widetilde{hfs}X^Ae)$.

Theorem 5.3. Let (HFS$\widetilde{XE}$, HFSτ) and (HFQ$\widetilde{YF}$, HFSτ*) be HF Soft topological spaces. Let f^{pu}: HFS$\widetilde{XE}$ → HFQ$\widetilde{YF}$ be a function. Then the following statements are equivalent.

(i) f^{pu} is HF Soft pu-continuous;

(ii) For each $\widetilde{hfm}X^B \in$ HFSτ*, $f^{-1}(\widetilde{hfm}X^B) \in$ HFSτ ;

(iii) For each HF Soft closed set $\widetilde{hfg}X^B$ over Y, $(f^{pu})^{-1}(\widetilde{hfg}X^B)$ is HF Soft closed over X.

Proof. To prove (i) ⇒(ii).

Let $\widetilde{hfm}X^B \in$ HFSτ* and $\widetilde{hfs}X^Ae \in (f^{pu})^{-1}(\widetilde{hfm}X^B)$.

We will show that $(f^{pu})^{-1}(\widetilde{hfm}X^B) \in N_{HFS\tau}(\widetilde{hfs}X^Ae)$.

Since $f^{pu}(\widetilde{hfs}X^Ae) \in \widetilde{hfm}X^B$ and $\widetilde{hfm}X^B \in$HFSτ*,

$\widetilde{hfm}X^{B} \in N_{HFS\tau^*}(f^{pu} (\widetilde{hfs}X^{A}e))$.
Since f^{pu} is HF Soft pu-continuous at $\widetilde{hfs}X^{A}e$, there exits $\widetilde{hft}X^{A} \in N_{HFS\tau}(\widetilde{hfs}X^{A}e)$
such that $f^{pu} (\widetilde{hft}X^{A}) \subseteq \widetilde{hfm}X^{B}$.
Therefore, we have $\widetilde{hfs}X^{A}e \in \widetilde{hft}X^{A} \subseteq (f^{pu})^{-1}(\widetilde{hfm}X^{B})$ and
so $(f^{pu})^{-1}(\widetilde{hfm}X^{B}) \in N_{HFS\tau}(\widetilde{hfs}X^{A}e)$.
To prove (ii) ⇒(iii).
Let $\widetilde{hfg}X^{B}$ be HF Soft closed over Y.
Then $\widetilde{hfg}X^{Bc} \in$ HFSτ* and by (ii), $(f^{pu})^{-1}(\widetilde{hfg}X^{Bc}) \in$ HFSτ .
Since $(f^{pu})^{-1}(\widetilde{hfg}X^{Bc}) = (f^{pu})^{-1}(\widetilde{hfg}X^{B})^{c}$,
we have that $(f^{pu})^{-1}(\widetilde{hfg}X^{B})$ is soft closed over X.
To prove (iii)⇒(ii).
It is similar to that of (ii) ⇒(iii).
To prove (ii) ⇒(i).
Let $\widetilde{hfs}X^{A}e \in \widetilde{hfs}X^{A}$ and $\widetilde{hfq}X^{B} \in N_{HFS\tau^*}(f^{pu} (\widetilde{hfs}X^{A}e))$.
Then there is a HF Soft open set $\widetilde{hfm}X^{B} \in$ HFSτ* such that
$f^{pu} (\widetilde{hfs}X^{A}e) \in \widetilde{hfm}X^{B} \subseteq \widetilde{hfq}X^{B}$.
By (ii), $(f^{pu})^{-1}(\widetilde{hfm}X^{B}) \in$ HFSτ and $\widetilde{hfs}X^{A}e \in (f^{pu})^{-1}(\widetilde{hfm}X^{B}) \subseteq (f^{pu})^{-1}(\widetilde{hfq}X^{B})$.
This shows that $(f^{pu})^{-1}\widetilde{hfq}X^{B} \in N_{HFS\tau}(\widetilde{hfs}X^{A}e)$.
Therefore, we have f^{pu} is HF Soft pu-continuous at every point $\widetilde{hfs}X^{A}e \in \widetilde{hfs}X^{A}$.

Theorem 5.4. Let (HFS$\widetilde{XE}$, HFSτ) and (HFQ$\widetilde{YF}$, HFSτ*) be HF Soft topological spaces. For a function f^{pu}: HFS$\widetilde{XE}$ → HFQ$\widetilde{YF}$, consider the following statements:
(i) f^{pu} is HF Soft pu-continuous;
(ii) For each HF Soft set $\widetilde{hfs}X^{A}$ over X, the inverse image of every neighbourhood of $f^{pu}(\widetilde{hfs}X^{A})$ is a neighbourhood of $\widetilde{hfs}X^{A}$;
(iii) For each HF Soft set $\widetilde{hfs}X^{A}$ over X and each neighbourhood $\widetilde{hfm}X^{B}$ of $f^{pu}(\widetilde{hfs}X^{A})$, there is a neighbourhood $\widetilde{hfq}X^{A}$ of $\widetilde{hfs}X^{A}$ such that $f^{pu}(\widetilde{hfq}X^{A}) \subseteq \widetilde{hfm}X^{B}$;
(iv) For each sequence { $\widetilde{hfg}nX^{A}$: n = 1, 2, ….} of HF Soft sets over X which
converges to a HF Soft set $\widetilde{hfs}X^{A}$ over X, the sequence { $f^{pu}(\widetilde{hfg}nX^{A})$: n = 1, 2,…..}
converges to $f^{pu}(\widetilde{hfs}X^{A})$.
Then we have (i) ⇔ (ii) ⇔ (iii) ⇔ (iv). Further, if the neighbourhood system of each HF Soft set over X is countable, then (iv) implies (i) and hence all of the above statements are equivalent.
Proof. To prove (i) ⇒(ii).
Let f^{pu} be HF Soft pu-continuous.
If $\widetilde{hfm}X^{B}$ is a neighbourhood of $f^{pu}(\widetilde{hfs}X^{A})$, then $\widetilde{hfm}X^{B}$ contains a HF Soft open neighbourhood $\widetilde{hfq}X^{B}$ of $f^{pu}(\widetilde{hfs}X^{A})$.
Since $f^{pu}(\widetilde{hfs}X^{A}) \subseteq (\widetilde{hfq}X^{B}) \subseteq \widetilde{hfm}X^{B}$,
$(f^{pu})^{-1}(f^{pu} (\widetilde{hfs}X^{A})) \subseteq (f^{pu})^{-1}(\widetilde{hfq}X^{B}) \subseteq (f^{pu})^{-1}(\widetilde{hfm}X^{B})$.
But $\widetilde{hfs}X^{A} \subseteq (f^{pu})^{-1}(f^{pu}(\widetilde{hfs}X^{A}))$ and $(f^{pu})^{-1}(\widetilde{hfq}X^{B})$ is HF Soft open.
Consequently, $(f^{pu})^{-1}(\widetilde{hfm}X^{B})$ is a neighbourhood of $\widetilde{hfs}X^{A}$.
To prove (ii) ⇒(i).
We use previous theorem. Let $\widetilde{hfq}X^{B}$ be HF Soft open over Y.
Then $(f^{pu})^{-1}\widetilde{hfq}X^{B}$ is a HF Soft subset of $\widetilde{hfs}X^{A}$.
Let $\widetilde{hfs}X^{A}$ be any HF Soft subset of $(f^{pu})^{-1}(\widetilde{hfq}X^{B})$.
Then $\widetilde{hfq}X^{B}$ is a HF Soft open neighbourhood of $f^{pu} (\widetilde{hfs}X^{A})$,
and by (ii), $(f^{pu})^{-1}(\widetilde{hfq}X^{B})$ is a HF Soft neighbourhood of $\widetilde{hfs}X^{A}$.
This shows that $(f^{pu})^{-1}(\widetilde{hfq}X^{B})$ is a HF Soft open set by known theorem.
To prove (ii) ⇒(iii).
Let $\widetilde{hfs}X^{A}$ be any HF Soft set over X and let $\widetilde{hfm}X^{B}$ be any neighbourhood of $f^{pu}(\widetilde{hfg}X^{A})$.
By (ii), $(f^{pu})^{-1}(\widetilde{hfm}X^{B})$ is a neighbourhood of $\widetilde{hfs}X^{A}$.
Then there exists a HF Soft open set $\widetilde{hfq}X^{A}$ in $\widetilde{hfs}X^{A}$
such that $\widetilde{hfs}X^{A} \subseteq \widetilde{hfq}X^{A} \subseteq (f^{pu})^{-1}(\widetilde{hfm}X^{B})$.
Thus, we have a HF Soft open neighbourhood $\widetilde{hfq}X^{A}$ of $\widetilde{hfs}X^{A}$
such that $f^{pu}(\widetilde{hfs}X^{A}) \subseteq f^{pu}(\widetilde{hfq}X^{A}) \subseteq \widetilde{hfm}X^{B}$.
To prove (iii) ⇒(ii).
Let $\widetilde{hfm}X^{B}$ be a neighbourhood of $f^{pu}(\widetilde{hfs}X^{A})$. Then there is a neighbourhood $\widetilde{hfq}X^{A}$ of $\widetilde{hfs}X^{A}$ such that $f^{pu}(\widetilde{hfq}X^{A}) \subseteq \widetilde{hfm}X^{B}$.
Hence $(f^{pu})^{-1}(f^{pu}(\widetilde{hfq}X^{A})) \subseteq (f^{pu})^{-1}(\widetilde{hfm}X^{B})$.
Furthermore, since $\widetilde{hfq}X^{A} \subseteq (f^{pu})^{-1}(f^{pu}(\widetilde{hfq}X^{A}))$,
$(f^{pu})^{-1}(\widetilde{hfm}X^{B})$ is a neighbourhood of $\widetilde{hfs}X^{A}$.
To prove (iii) ⇒(iv).
If $\widetilde{hfm}X^{B}$ is a neighbourhood of $f^{pu}(\widetilde{hfs}X^{A})$, there is a neighbourhood $\widetilde{hfq}X^{A}$ of $\widetilde{hfs}X^{A}$ such that $f^{pu}(\widetilde{hfq}X^{A}) \subseteq \widetilde{hfm}X^{B}$.

Since {($\widetilde{hfg}$nX^A) : n = 1, 2,...} is eventually in $\widetilde{hfq}X^A$,
we have f^{pu}($\widetilde{hfg}$nX^A) ⊆ f^{pu}($\widetilde{hfq}X^A$) ⊆ $\widetilde{hfm}$nX^B for n ≥ m;
i.e., there is an m such that for n ≥ m, $\widetilde{hfg}$nX^A ⊆ ($\widetilde{hfq}X^A$).
Therefore, {f^{pu}($\widetilde{hfg}$nX^A) : n = 1, 2, ...} converges to f^{pu}($\widetilde{hfs}X^A$).
To prove (iv) ⇒(i).
Suppose that the neighbourhood system of each HF Soft set over X is countable.
Let $\widetilde{hfq}X^B$ be any HF Soft open set over Y.
Then $(f^{pu})^{-1}$($\widetilde{hfq}X^B$) is a HF Soft subset of $\widetilde{hfs}X^A$.
Let $\widetilde{hfs}X^A$ be any HF Soft subset of $(f^{pu})^{-1}$($\widetilde{hfq}X^B$), and
let $\widetilde{hfg}$1X^A, $\widetilde{hfg}$2X^A, ..., $\widetilde{hfg}$nX^A, ...be the neighbourhood system $\widetilde{hfs}X^A$.
Let $\widetilde{hfm}$nX^A = $\cap_{i=1 \text{ to } n}$($\widetilde{hfg}$iX^A).
Then $\widetilde{hfm}$1X^A, $\widetilde{hfm}$2X^A, ... , $\widetilde{hfm}$nX^A ..... is a sequence which is eventually contained in each neighbourhood of $\widetilde{hfs}X^A$,
i.e., $\widetilde{hfm}$1X^A, $\widetilde{hfm}$2X^A, ... , $\widetilde{hfm}$nX^A .... converges to $\widetilde{hfs}X^A$.
Hence, there is an m such that for n ≥ m, $\widetilde{hfm}$nX^A ⊆ $(f^{pu})^{-1}$($\widetilde{hfq}X^B$).
Since for each n, $\widetilde{hfm}$nX^A is a neighbourhood of $\widetilde{hfs}X^A$,
$(f^{pu})^{-1}$($\widetilde{hfq}X^B$) is a neighbourhood of $\widetilde{hfs}X^A$.
This shows that $(f^{pu})^{-1}$($\widetilde{hfq}X^B$) is HF Soft open.

Theorem 5.5. Let f^{pu} be a HF Soft pu-continuous function carrying the HF Soft compact topological space (HFS$\widetilde{XE}$, HFSτ) onto the HF Soft topological space (HFQ$\widetilde{YF}$, HFSτ*) Then (HFQ$\widetilde{YF}$, HFSτ*) is HF Soft compact.
Proof. Let {($\widetilde{hfq}$iX^B): i ∈ I} be a cover of HFQ$\widetilde{YF}$ by HF Soft open sets.
Then since f^{pu} is HF Soft pu-continuous,
the family of all HF Soft sets of the form $(f^{pu})^{-1}$($\widetilde{hfq}$iX^B), for ($\widetilde{hfq}$iX^B) ∈ {($\widetilde{hfq}$iX^B) : i ∈ I}, is a HF Soft open cover of $\widetilde{hfs}X^A$ which has a HF Soft finite sub cover.
Since f^{pu} is surjective, it can be is easily verified that f^{pu}(($f^{pu})^{-1}$($\widetilde{hfq}X^B$)) = $\widetilde{hfq}X^B$ for any HF Soft set $\widetilde{hfq}X^B$ over Y.
Thus, the class of images of members of the HF Soft sub cover is a finite sub family of which covers HFQ$\widetilde{YF}$.
Hence (HFQ$\widetilde{YF}$, HFSτ*) is HF Soft compact.

Defnition 5.6. Let (HFS$\widetilde{XE}$, HFSτ) and (HFQ$\widetilde{YF}$, HFSτ*) be two HF Soft topological spaces.
A HF Soft function $\widetilde{hfs}$: HFS$\widetilde{XE}$ → HFQ$\widetilde{YF}$ is called HF Soft closed if $\widetilde{hfs}$ ($\widetilde{hfg}X^A$)
is HF Soft closed set in HFQ$\widetilde{YF}$, for all HF Soft closed sets $\widetilde{hfg}X^A$ in HFS$\widetilde{XE}$.

Theorem 5.7. Let (HFS$\widetilde{XE}$, HFSτ)be a HF Soft topological space and (HFQ$\widetilde{YF}$, HFSτ*) be a HF Soft Hausdorff space. HF Soft function $\widetilde{hfs}$ is closed if HF Soft function $\widetilde{hfs}$: HFS$\widetilde{XE}$ → HFQ$\widetilde{YF}$ is continuous.
Proof. Let $\widetilde{hfq}X^B$ be any HF Soft closed set in HFS$\widetilde{XE}$.
By known theorem we know that $\widetilde{hfq}X^B$ is compact.
Since HF Soft function $\widetilde{hfs}$ is continuous, HF Soft set $\widetilde{hfs}$($\widetilde{hfq}X^B$) is compact in HFQ$\widetilde{YF}$.
As HFS$\widetilde{YF}$ is HF Soft Hausdorff space, HF Soft set $\widetilde{hfs}$($\widetilde{hfq}X^B$) is closed.
Thus HF Soft function $\widetilde{hfs}$ is closed.

6. *CONCLUSION*

In this paper, we have defined HF Soft T0- Space, HF Soft T1- Space HF Soft T2- Space or HF Soft Hausdorff space, HF Soft regular space, HF Soft T3- Space, HF Soft normal space and HF Soft T4- Space and also studied countable HF Soft basis, first countable HF Soft space and second countable HF Soft space. Finally we have given the separation axioms, defined HF Soft connected space then HF Soft pu continuous mapping, HF Soft pu open mapping and HF Soft pu closed mapping and studied various properties. In future work we consider the study and definition of HF Soft metric space. These studies will play a significant role in the development of hesitant fuzzy soft topology.

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