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A SIMPLE STUDY: GENERATION OF MANDELBROT SET IN DYNAMICS

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Abstract:

The purpose of this review paper is to study the simple way of generating mandelbrot set in a graph-

ical platform. This review paper initially conducts with the construction theory like fixed point theo- rem. Then this paper explains the simple way to generate Mandelbrot set.

Key words:
Mathematical visualiztion, Dynamics system, fixed point theorem, Camplex plane, Itera- tion

INTRODUCTION:

The Mandelbrot set is the set of values of c in the complex plane for which the orbit of 0 under iteration of the complex quadratic polynomial $z_{n+1} = z_n^2 + c$ remains bounded. That is, a complex number c is part of the Mandelbrot set if, when starting with $z_0 = 0$ and applying the iteration repeatedly, the absolute value of z_n remains bounded however large n gets.

For example, letting $c = 1$ gives the sequence $0,1,2,5,26,\ldots$ which tends to infinity. As this sequence is unbounded, 1 is not an element of the Mandelbrot set. On the other hand, $c = i$ (where i is $i^2 = -1$) gives the sequence $0, i, (-1+i), (-1+i), -i, \ldots$ which is bounded, and so i belongs to the Mandelbrot set.[1]

THE FIXED POINT THEORY:

One of the simplest criteria for finding fixed points is an immediate consequence of the following important fact from calculus. [2]

The Intermediate Value Theorem: Suppose $\mathbf{F}: [a, b] \rightarrow \mathbf{R}$

Suppose y_0 lies between $\mathbf{F}(a)$ *and* $\mathbf{F}(b)$. Then there is an x_0 in the internal $[a, b]$ with $\mathbf{F}(x_0) = y_0$.

Simply stated, this theorem tells us that a continuous function assumes all values between $\mathbf{F}(a)$ and $\mathbf{F}(b)$ on the interval $[a, b]$. An immediate consequence is:

Fixed Point theorem:

Suppose $\mathbf{F}: [a, b] \rightarrow [a, b]$ is continuous. Then there is a fixed point for $\mathbf{F}$ in $[a, b]$.

Remarks:

1. This theorem asserts this existence of at least one fixed point for $\mathbf{F}$ in [a, b]; there may of course be more. For example, all points in any interval $[a, b]$ are fixed by the identity function $\mathbf{F}(x) = x$.
2. There are several important hypotheses in this theorem, the first two being continuity and the fact that $\mathbf{F}$ takes the intervals $[a, b]$ into itself. Violation of either of these may yield a function without fixed points.
3. It is important that the interval $[a, b]$ be closed. For example, $\mathbf{F}(x) = x^2$ takes the interval $\left(\frac{0,1}{2}\right)$ inside itself and is continuous, but there are no fixed points in this open interval.
4. While the fixed point theorem asserts the existence of at least one fixed point, it unfortunately does not give us any method of actually finding this point. However, in practice, we often don't need to know exactly where the fixed point lies. Just the knowledge that it is present in a certain interval often suffices for our purposes.

The proof of the fixed point Theorem follows from the Intermediate Value Theorem applied to $H(x) = F(a) - a \geq \mathbf{0}$

$$H(b) = \mathbf{F}(b) - b \leq \mathbf{0}$$

Thus there is a c in the interval [a, b] with $H(c) = 0.$ this c satisfies $\mathbf{F}(c) - c = \mathbf{0}$ and so is our fixed point [2]

MANDELBROT SET GENERATION:

The Mandelbrot set is the visual representation of an iterated function on the complex plane. To generate a Mandelbrot set we need to know about iteration.

Iteration means,

INPUT FUNCTION OUTPUT

If a iterated function is

$$f(x) = x + 1$$
$$f(2) = 2 + 1 = 3$$
$$f(3) = 3 + 1 = 4$$

In iterated function the output of a function is used as input in the next iteration. To generate Mandelbrot set we have work with this function.

When iterating we use c as the first input.

$$F(x) = x^2 + c$$

$$F(2) = 4 + 2 = 6$$

$$F(6) = 36 + 2 = 38$$

$$F(38) = 1444 + 2 = 1446$$

$$F(1446) = 2090916 + 2 = 2090918$$

Here the iteration output is growing exponentially so in the number line the output will be,

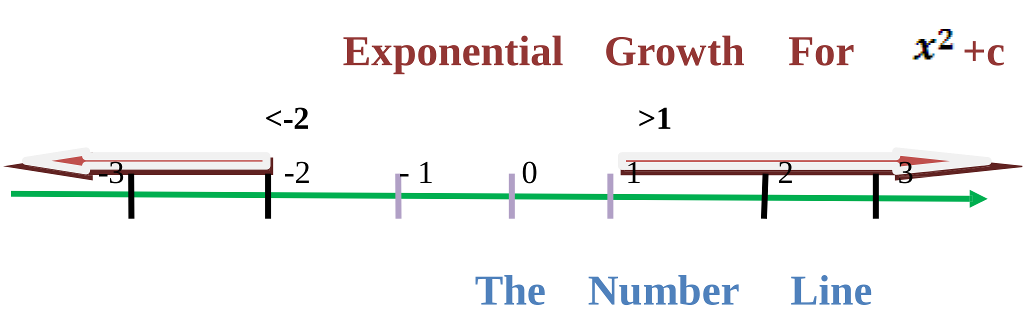

Fig1. Number line

As the exponential growth is in the number line is greater than 1 and less than -2, a weird thing happens between -2 and 1. And the Mandelbrot set is generated on this region.[3]

If we take a input of fraction number then

$F(x) = x^2 + c$

$F(0.1) = 0.11$

$F(0.11) = 0.1121$

$F(0.1121) = 0.1125641$

$F(0.1125641) = 0.112671196$

So between -2 and 0 the iteration function will act differently .In this region if we take -1 as input than ,

$f(-1) = 1 + (-1) = 0$

$f(0) = 0 + (-1) = -1$

$f(-1) = 1 + (-1) = 0$

$f(0) = 0 + (-1) = -1$

Again if we take -2 as the first input in function $f(x) = x^2 + c$

$f(-2) = 4 + (-2) = 2$

$f(2) = 4 + (-2) = 2$

$f(2) = 4 + (-2) = 2$

$f(2) = 4 + (-2) = 2$

So in the region of -2 and 0 the output is also scattering between -2 and 0. This is called chaotic behavior.[3]

Now if we think about a complex plane, for some of the input output will be beyond the region of 2and -2.

We take one iteration for a complex number 1+i then,

$f(x) = x^2 + c$

$f(x) = (1+i)^2 + (1+i) = (1+3i)$

We can see taking input $1+i$ the output is $(1+3i)$ which is beyond the region in the complex plan.

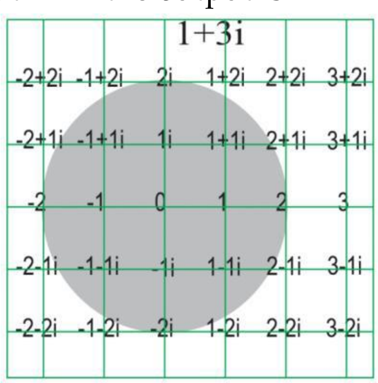

Fig2.Showning the Mandelbrot set outline in complex plane

So after first iteration all number within the circle which's output are escaped from the circle will be colored as red so we can get the image like this,

Fig3. Mandelbrot set outline after first iteration

If we continue this iterating up to 20 iteration we can get a image like below which is called Mandelbrot set.

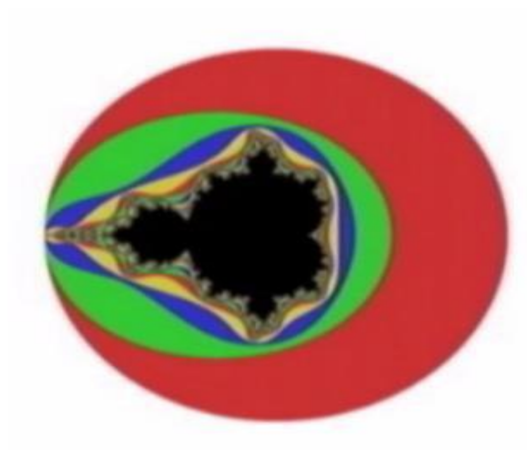

Fig4: Mandelbrot set

Conclusion:

This review paper work named "A simple study: generation of Mandelbrot set in dynamics" is based on the different aspects on Mandelbrot set in dynamics which is an important part of the Universal algebra. It has given better understanding of function and specially, Mandelbrot set and its generation. We have made all efforts to represent the fundamental concepts of Mandelbrot set in dynamics.

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