



# THEORY OF ELECTRONIC RAMAN SCATTERING IN HIGH TEMPERATURE SUPERCONDUCTORS

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## Abstract

A novel ab-initio theory is investigated to obtain the Raman intensity for a high temperature superconductor. The theory is developed using the many body theory for electrons and phonons with the help of an almost complete newly formulated Hamiltonian which involves the interactions of (i) bare electrons, (ii) harmonic phonons, (iii) anharmonic phonons, (iv) electrons and phonons and (v) localized phonons. This work based on quantum dynamics of electron describes that the electron-phonon coupling coefficient inevitably shows its presence in all sectors of phonon fields namely, the field of anharmonic phonons, localized phonons and interfering phonons. In this new framework, the electronic Raman scattering, spectral density function has been obtained and analyzed for high temperature superconductors for the first time in two ways: (i) in the close vicinity of  $T_c$  the superconductivity increases and (ii) the contribution of normal energy becomes small compared to the cooper pair energy. The temperature and defect dependencies are discussed in detail along with the nature of continues and line spectra. The general temperature dependence of Raman tensor is also established.

**Key words:** Cooper Pair, Model Hamiltonian, Raman scattering, differential cross section, Electronic Raman scattering

## 1 Introduction

When light is scattered from a molecule or a solid, most of the photons are elastically scattered. The scattered photons have the same energy as the incident photons. However, a small fraction of light is scattered as optical energies, different from the energy of the incident photons. The process leading to this inelastic scattering is termed as Raman Effect [1]. The difference in energy between the incident photon and the Raman scattered photon is equal to the energy of a vibration of the molecule or lattice vibration of solid sample. The plot of intensity of scattered light versus energy difference is a Raman spectrum. In fact, Raman Effect is the inelastic scattering of light by the lattice vibration due to the fluctuations in the crystal electronic polarizability induced by the displacement of atoms from its equilibrium position [2-4]. The part of scattered light with lower frequency than the incident light is called the Stokes component, while the part of the scattered light with higher frequency is called the anti-Stokes component.

The system can be quantized and each quantum of normal mode is known as a “phonon”. The multi-phonon interactions aroused a great deal of interest in the calculation of thermal, optical and elastic properties of crystals. With the special reference to crystalline solids, it is seen that most of purposed theories are based on the harmonic approximation. Actually we should not expect the harmonic theory to be correct, even at the lowest temperatures. Because of the influence of anharmonicities does not vanish even at the absolute temperature. In addition to anharmonicity the presence of impurity dramatically modifies the phonon spectrum of the crystal and, hence, substantial changes occur in energy dependent properties of the crystal. The interaction such as: (i) phonon interaction in the anharmonic crystal fields, (ii) phonon interactions in the localized fields of crystal impurity, (iii) anharmonic phonon interactions with localized phonons when phonons of one field approach the phonons of another field or vice versa, and (iv) electron-phonon interactions. These interactions give rise to anharmonic mode, impurity modes and impurity-anharmonicity interference modes. The involvement of anharmonic and impurity effects can cause considerable alternation in the phonon Raman intensity. In the present work, we shall apply the powerful Green’s function technique to investigate the Raman intensity in high temperature superconductors. This thermodynamic Green’s function technique has become an invaluable tool in the study of complicated systems of interacting particles. In case of Raman line both the Raman shifts and line widths, and also the scattered intensities vary with temperature. The variation with temperature of the shifts and widths of Raman lines has been measured by many authors in different crystals.

Since the discovery of high temperature superconductors Raman scattering has proven to be an excellent technique to characterize them and to investigate basic physical properties relevant to the elusive mechanism responsible for the phenomenon of superconductivity. However, in the field of high temperature superconductors many of their properties have not yet been understood. In particular, the symmetry of the superconducting gap is still controversial [1-3]. A wide range of experimental techniques can be employed to investigate the properties of the gap function for which the Raman scattering has played an important role [4, 5]. The measurements of Raman spectra have been a valuable tool in the investigation of vibrational and rotational energy levels of molecules and in the determination of lattice vibrational frequencies of crystalline solids.

The temperature dependent electron energies and line widths can only be described in terms of the anharmonic character of lattice vibrations. It is the crystal anharmonicity which can successfully explain the origin and nature of second order Raman scattering and third-order Raman scattering processes

## 2 Raman Scattering

### 2.1 Raman Tensor

The Raman scattering tensor  $i_{\alpha\gamma,\beta\lambda}(\in_R)$  is given by[7]

$$i_{\alpha\gamma,\beta\lambda}(\in_R) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} dt \exp(-i \in_R t) \langle P_{\beta\lambda}(U, t) P_{\alpha\gamma}(U, 0) \rangle_T \quad (1)$$

where the subscript  $T$  stands for the time ordering and  $P_{\beta\lambda}(U, t)$  is the electronic polarizability of the crystal in a fixed nuclear configuration  $U$  and in its ground electronic state; the brackets denote a thermal average over nuclear configurations. The electronic polarizability for  $N$  cells can be expanded in a Taylor series which is dependent on the normal coordinates  $U_j^{(k)}(t)$  of the crystal, in the form [7]

$$P_{\beta\lambda}(t) = P_{\beta\lambda}^{(0)} + N^{1/2} \sum_{k_1 j_1} P_{\beta\lambda}^{(k_1)} U_{j_1}^{(k_1)}(t) + 1/2! \sum_{\substack{k_1 j_1 \\ k_2 j_2}} P_{\beta\lambda}^{(k_1 k_2)} U_{j_1}^{(k_1)}(t) U_{j_2}^{(k_2)}(t) \\ + \sum_{\substack{k_1 j_1, \\ k_2 j_2, k_3 j_3}} P_{\beta\lambda}^{(k_1 k_2 k_3)} U_{j_1}^{(k_1)}(t) U_{j_2}^{(k_2)}(t) U_{j_3}^{(k_3)}(t) + \dots \quad (2)$$

The symbols appearing in Eq.(2) are well discussed in the references elsewhere [7,9,10]. In this equation first term contributes to Rayleigh scattering, the second term to one phonon Raman scattering (FORS), the next second-order Raman scattering (SORS), and so on. We shall neglect first term from above Eq.(2) because we are not interested in Rayleigh scattering and the remaining terms are

$$P_{\beta\lambda}^{(k_1)} = \sum_{k_1 j_1, \gamma} P_{\beta\lambda, \gamma}^{(k_1)}(j_1, l_1) (m_{k_1})^{-1/2} e_{\gamma}^{(k_1)} \exp[2\pi i(\mathbf{k}_1 \cdot \mathbf{R}_{l_1})] \quad (3a)$$

$$P_{\beta\lambda}^{(k_1 k_2)} = \sum_{\substack{k_1 j_1, k_2 j_2 \\ \gamma \delta, l_1}} P_{\beta\lambda, \gamma \delta}^{(k_1 k_2)}(j_1 j_2, l_1 l_2) (m_{k_1} m_{k_2})^{-1/2} e_{\gamma}^{(k_1)} e_{\delta}^{(k_2)} \exp[2\pi i(\mathbf{k}_1 \cdot \mathbf{R}_{l_1} + \mathbf{k}_2 \cdot \mathbf{R}_{l_2})] \quad (3b)$$

and

$$P_{\beta\lambda}^{(k_1 k_2 k_3)} = \sum_{\substack{k_1 j_1, k_2 j_2, \\ k_3 j_3, \gamma \delta \sigma, l_1 l_2}} P_{\beta\lambda, \gamma \delta \sigma}^{(k_1 k_2 k_3)}(j_1 j_2 j_3, l_1 l_2 l_3) (m_{k_1} m_{k_2} m_{k_3})^{-1/2} e_{\gamma}^{(k_1)} e_{\delta}^{(k_2)} e_{\sigma}^{(k_3)} \\ \times \exp[2\pi i(\mathbf{k}_1 \cdot \mathbf{R}_{l_1} + \mathbf{k}_2 \cdot \mathbf{R}_{l_2} + \mathbf{k}_3 \cdot \mathbf{R}_{l_3})] \quad (3c)$$

With 
$$P_{\beta\lambda, \gamma}^{(k_1)}(j_1, l_1) = \left[ \frac{\partial P_{\beta\lambda}}{\partial U_{\gamma}^{(k_1)}(j_1)} \right]_0 \quad (4a)$$

$$P_{\beta\lambda, \gamma \delta}^{(k_1 k_2)}(j_1 j_2, l_1 l_2) = \left[ \frac{\partial^2 P_{\beta\lambda}}{\partial U_{\gamma}^{(k_1)}(j_1) \partial U_{\delta}^{(k_2)}(j_2)} \right]_0 \quad (4b)$$

$$P_{\beta\lambda, \gamma \delta \sigma}^{(k_1 k_2 k_3)}(j_1 j_2 j_3, l_1 l_2 l_3) = \left[ \frac{\partial^3 P_{\beta\lambda}}{\partial U_{\gamma}^{(k_1)}(j_1) \partial U_{\delta}^{(k_2)}(j_2) \partial U_{\sigma}^{(k_3)}(j_3)} \right]_0 \quad (4c)$$

and so on, defined as coefficients of expansion. In Eq.(3)  $e$ 's are the eigenvectors of the Fourier-Transformed dynamical matrix,  $m_k$  is the mass of the  $k^{th}$  type of atom,  $R_l$  are the position vectors of the  $l^{th}$  atom in the

undistorted lattice. The various Raman tensors can be obtained on substitution of Eq.(2) in Eq.(2) followed by second quantized normal coordinate transformation yields

$$i_{\alpha\gamma,\beta\lambda}(\epsilon_R) = i_{\alpha\gamma,\beta\lambda}^{(1)}(\epsilon_R) + i_{\alpha\gamma,\beta\lambda}^{(2)}(\epsilon_R) + i_{\alpha\gamma,\beta\lambda}^{(3)}(\epsilon_R) \quad (5)$$

$$i_{\alpha\gamma,\beta\lambda}^{(1)}(\epsilon_R) = \frac{1}{2\pi} \sum_{k_1j_1, k_1'j_1'} \int_{-\infty}^{+\infty} dt \exp(-i \epsilon_R t) P_{\alpha\gamma,\beta\lambda}^{(1)} \left( \begin{matrix} k_1 & k_1' \\ j_1 & j_1' \end{matrix} \right) \langle A_{k_1j_1}(t) A_{k_1'j_1'}(0) \rangle \quad (6a)$$

$$i_{\alpha\gamma,\beta\lambda}^{(2)}(\epsilon_R) = \frac{1}{2\pi} \sum_{\substack{k_1j_1, k_1'j_1' \\ k_2j_2, k_2'j_2'}} \int_{-\infty}^{+\infty} dt \exp(-i \epsilon_R t) P_{\alpha\gamma,\beta\lambda}^{(2)} \left( \begin{matrix} k_1 & k_2 & k_1' & k_2' \\ j_1 & j_2 & j_1' & j_2' \end{matrix} \right) \langle A_{k_1j_1}(t) A_{k_2j_2}(t) A_{k_1'j_1'}(0) A_{k_2'j_2'}(0) \rangle \quad (6b)$$

and

$$i_{\alpha\gamma,\beta\lambda}^{(3)}(\epsilon_R) = \frac{1}{2\pi} \sum_{\substack{k_1j_1, k_2j_2, k_3j_3 \\ k_1'j_1', k_2'j_2', k_3'j_3'}} \int_{-\infty}^{+\infty} dt \exp(-i \epsilon_R t) P_{\alpha\gamma,\beta\lambda}^{(3)} \left( \begin{matrix} k_1 & k_2 & k_3 & k_1' & k_2' & k_3' \\ j_1 & j_2 & j_3 & j_1' & j_2' & j_3' \end{matrix} \right) \\ \times \langle A_{k_1j_1}(t) A_{k_2j_2}(t) A_{k_3j_3}(t) A_{k_1'j_1'}(0) A_{k_2'j_2'}(0) A_{k_3'j_3'}(0) \rangle \quad (6c)$$

In obtaining Eq.(5) and (6) we have ignored the contributions from equal time correlation functions [6]. The various coefficients appearing in Eq.(6) are given by

$$P_{\alpha\gamma,\beta\lambda}^{(1)} \left( \begin{matrix} k_1 & k_1' \\ j_1 & j_1' \end{matrix} \right) = (\hbar / 2M) \Delta(\mathbf{k}_1 + \mathbf{k}_1') e^{(k_1)}_{j_1} e^{(k_1')}_{j_1'} P_{\beta\lambda,\gamma} \left( \begin{matrix} k_1 & l_1 \\ j_1 & j_1' \end{matrix} \middle| \begin{matrix} k_1' & l_1' \\ j_1' & j_1' \end{matrix} \right) \\ \times \frac{\exp[2\pi i(\mathbf{k}_1 + \mathbf{k}_1') \cdot \mathbf{R}_{l_1}]}{[\epsilon \left( \begin{matrix} k_1 \\ j_1 \end{matrix} \right) \epsilon \left( \begin{matrix} k_1' \\ j_1' \end{matrix} \right)]^{1/2}} \quad (7a)$$

$$P_{\alpha\gamma,\beta\lambda}^{(2)} \left( \begin{matrix} k_1 & k_2 & k_1' & k_2' \\ j_1 & j_2 & j_1' & j_2' \end{matrix} \right) = (\hbar / 4MN)^2 \Delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_1' + \mathbf{k}_2') e^{(k_1)}_{j_1} e^{(k_2)}_{j_2} e^{(k_1')}_{j_1'} e^{(k_2')}_{j_2'} \\ \times P_{\beta\lambda,\gamma\delta} \left( \begin{matrix} k_1 & k_2 \\ j_1 & j_2 \end{matrix} \middle| \begin{matrix} k_1' & k_2' \\ j_1' & j_2' \end{matrix} \right), l_1 l_2 \left( \begin{matrix} k_1' & k_2' \\ j_1' & j_2' \end{matrix} \middle| \begin{matrix} l_1' & l_2' \\ j_1' & j_2' \end{matrix} \right) \\ \times \frac{\exp[2\pi i\{(\mathbf{k}_1 + \mathbf{k}_1') \cdot \mathbf{R}_{l_1} + (\mathbf{k}_2 + \mathbf{k}_2') \cdot \mathbf{R}_{l_2}\}]}{[\epsilon \left( \begin{matrix} k_1 \\ j_1 \end{matrix} \right) \epsilon \left( \begin{matrix} k_2 \\ j_2 \end{matrix} \right) \epsilon \left( \begin{matrix} k_1' \\ j_1' \end{matrix} \right) \epsilon \left( \begin{matrix} k_2' \\ j_2' \end{matrix} \right)]^{1/2}} \quad (7b)$$

and

$$P_{\alpha\gamma,\beta\lambda}^{(3)} \left( \begin{matrix} k_1 & k_2 & k_3 & k_1' & k_2' & k_3' \\ j_1 & j_2 & j_3 & j_1' & j_2' & j_3' \end{matrix} \right) = (\hbar / 2MN)^3 \Delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 + \mathbf{k}_1' + \mathbf{k}_2' + \mathbf{k}_3') e^{(k_1)}_{j_1} e^{(k_2)}_{j_2} e^{(k_3)}_{j_3} \\ \times e^{(k_1')}_{j_1'} e^{(k_2')}_{j_2'} e^{(k_3')}_{j_3'} P_{\beta\lambda,\gamma\delta\sigma} \left( \begin{matrix} k_1 & k_2 & k_3 \\ j_1 & j_2 & j_3 \end{matrix} \middle| \begin{matrix} k_1' & k_2' & k_3' \\ j_1' & j_2' & j_3' \end{matrix} \right), l_1 l_2 l_3 \left( \begin{matrix} k_1' & k_2' & k_3' \\ j_1' & j_2' & j_3' \end{matrix} \middle| \begin{matrix} l_1' & l_2' & l_3' \\ j_1' & j_2' & j_3' \end{matrix} \right) \\ \times \frac{\exp[2\pi i\{(\mathbf{k}_1 + \mathbf{k}_1') \cdot \mathbf{R}_{l_1} + (\mathbf{k}_2 + \mathbf{k}_2') \cdot \mathbf{R}_{l_2} + (\mathbf{k}_3 + \mathbf{k}_3') \cdot \mathbf{R}_{l_3}\}]}{[\epsilon \left( \begin{matrix} k_1 \\ j_1 \end{matrix} \right) \epsilon \left( \begin{matrix} k_2 \\ j_2 \end{matrix} \right) \epsilon \left( \begin{matrix} k_3 \\ j_3 \end{matrix} \right) \epsilon \left( \begin{matrix} k_1' \\ j_1' \end{matrix} \right) \epsilon \left( \begin{matrix} k_2' \\ j_2' \end{matrix} \right) \epsilon \left( \begin{matrix} k_3' \\ j_3' \end{matrix} \right)]^{1/2}} \quad (7c)$$

where

$$P_{\beta\lambda,\gamma\dots} \left( \begin{matrix} k_1 & k_2 \\ j_1 & j_2 \end{matrix} \dots \middle| \begin{matrix} k_1' & k_2' \\ j_1' & j_2' \end{matrix} \dots \right) = P_{\beta\lambda,\gamma\dots} \left( \begin{matrix} k_1 & k_2 \\ j_1 & j_2 \end{matrix} \right) P_{\beta\lambda,\gamma\dots} \left( \begin{matrix} k_1' & k_2' \\ j_1' & j_2' \end{matrix} \right) \dots \quad (8)$$

The correlation functions  $\langle A_{k_1j_1}(t) \dots A_{k_1'j_1'}(0) \dots \rangle$  appearing in Eq.(6) contain the entire physics of Raman scattering. These correlation functions can be evaluated from [11-15]

$$F_{kj k'j'}^{(i)}(t, t') = \int_{-\infty}^{+\infty} J_{kj k'j'}^{(i)}(\epsilon) \exp[-i \epsilon (t - t')] d \epsilon \quad (9)$$

where

$$F_{kj k'j'}^{(i)}(t, t') = \langle A_{k_1j_1}(t) A_{k_2j_2} \dots (t) A_{k_i j_i}(t) A_{k_1'j_1'}(0) A_{k_2'j_2'} \dots (0) A_{k_i'j_i'}(0) \rangle \quad (10)$$

and  $J_{kj k'j'}^{(i)}(\epsilon)$  are the known as spectral density functions, given by

$$J_{kj k'j'}^{(i)}(\epsilon) = -\{2 / \exp(\beta \hbar \epsilon - 1)\} \text{Im} G_{kj}^{(i)}(\epsilon) \quad (11)$$

## 2.2 Differential Cross Section for Raman Scattering

The differential scattering cross section per unit solid angle per unit energy interval for Raman scattering is given by [6, 16 and 17]

$$\frac{d^2\sigma}{d\Omega d\epsilon_R} = (\epsilon/c)^4 \sum_{\alpha\beta,\gamma\lambda} n_\alpha n_\beta i_{\alpha\gamma,\beta\lambda}(\epsilon_R) m_\gamma m_\beta \quad (12)$$

where  $\Omega$  denotes solid angle,  $d\epsilon$  is solid angle located in the direction  $\epsilon$  and  $n$  and  $m$  are the unit polarization vectors of the radiation. Obviously, the differential cross section depends on the Raman intensity tensor  $i_{\alpha\gamma,\beta\lambda}(\epsilon_R)$ , which will describe the one-, two-, and three-phonon differential cross section for the Raman scattering.

## 3 The Hamiltonian

The almost complete Hamiltonian of such a system can be written as

$$H = H_p + H_e + H_{ep} + H_A + H_D \quad (13)$$

where

$$H_p = \sum_{\mathbf{k}} \frac{\epsilon_{\mathbf{k}}}{4} [A_{\mathbf{k}}^* A_{\mathbf{k}} + B_{\mathbf{k}}^* B_{\mathbf{k}}] \quad (\text{unperturbed phonon Hamiltonian}) \quad (14a)$$

$$H_e = \sum_{\mathbf{q}} \epsilon_{\mathbf{q}} b_{\mathbf{q}}^* b_{\mathbf{q}}, \quad (\text{unperturbed electron Hamiltonian}) \quad (14b)$$

$$H_{ep} = \sum_{\mathbf{k},\mathbf{q}} g_{\mathbf{k}} b_{\mathbf{q}}^* b_{\mathbf{q}} B_{\mathbf{k}}, \quad (\text{electron-phonon Hamiltonian}) \quad (14c)$$

$$H_A = \sum_{s \geq 3} \sum_{\mathbf{k}_1, \dots, \mathbf{k}_s} V_s(\mathbf{k}_1, \mathbf{k}_2, \dots, \mathbf{k}_s) A_{\mathbf{k}_1} A_{\mathbf{k}_2} \dots A_{\mathbf{k}_s} \quad (\text{anharmonic Hamiltonian}) \quad (14d)$$

and

$$H_D = \sum_{\mathbf{k}_1, \mathbf{k}_2} [D(\mathbf{k}_1, \mathbf{k}_2) A_{\mathbf{k}_1} A_{\mathbf{k}_2} - C(\mathbf{k}_1, \mathbf{k}_2) B_{\mathbf{k}_1} B_{\mathbf{k}_2}] \quad (\text{defect Hamiltonian}) \quad (14e)$$

In above expressions  $A_{\mathbf{k}} = a_{\mathbf{k}} + a_{-\mathbf{k}}^* = +A_{-\mathbf{k}}^*$  (phonon field operator) and  $B_{\mathbf{k}} = a_{\mathbf{k}} - a_{-\mathbf{k}}^* = -B_{-\mathbf{k}}^*$  (phonon momentum operator),  $b_{\mathbf{q}}$  ( $b_{\mathbf{q}}^*$ ) and  $a_{\mathbf{k}}$  ( $a_{\mathbf{k}}^*$ ) are electron and phonon annihilation (creation) operators with wave vectors  $\mathbf{q}$  and  $\mathbf{k}$  respectively.  $\mathbf{Q} = \mathbf{k} + \mathbf{q}$ .  $\epsilon_{\mathbf{k}}$ ,  $\epsilon_{\mathbf{q}}$ ,  $g_{\mathbf{k}}$ ,  $V_s(\mathbf{k}_1, \mathbf{k}_2, \dots, \mathbf{k}_s)$ ,  $C(\mathbf{k}_1, \mathbf{k}_2)$  and  $D(\mathbf{k}_1, \mathbf{k}_2)$  stand for the electron energy, phonon energy and electron-phonon coupling coefficient, anharmonic coefficients, mass difference and force constant change parameters, respectively.

## 4 Electron Green's function

Let us consider the double-time thermodynamic electron retarded Green's functions

$$G_{qq'}(t, t') = \langle \langle b_{q\sigma}^*(t); b_{q'\sigma'}(t') \rangle \rangle = -i\theta(t-t') \langle [b_{q\sigma}^*, b_{q'\sigma'}] \rangle \quad (15)$$

In above Eq.(15),  $\sigma$  defines the spin and  $\uparrow$  ( $\downarrow$ ) designates the spin up (down) for electrons.

The Green's functions can be evaluated with the help of equation of motion technique of quantum dynamics via Hamiltonian (14) by adopting the technique of successive approximations and some tedious algebra as of the form

$$G_{qq'}(\epsilon) = \frac{(3\epsilon^N + \epsilon^C) \delta_{qq'} \delta_{\sigma\sigma'}}{\{2\pi[\epsilon^2 - \tilde{\epsilon}_{q\sigma}^2 + (3\epsilon^N + \epsilon^C)P(q, \epsilon)]\}} \quad (16)$$

where  $\tilde{\epsilon}_{q\sigma}^2$  ( $\equiv \tilde{\epsilon}_q^2$ ) is the renormalized energy, which is equal to

$$\tilde{\epsilon}_q^2 = (3\epsilon^N + \epsilon^C)^2 - \frac{X_1}{2\pi} - \frac{1}{(3\epsilon^N + \epsilon^C)} \sum_{q'} (g_k + g_{k'}) \frac{X_2}{2\pi} \quad (17)$$

with  $X_1 = \langle [F_{q\sigma}^*(t), b_{q'\sigma'}] \rangle$  and  $X_2 = \langle [F_{q\sigma}^*(t), b_{q'\sigma'} B_{k'}] \rangle$   
 and

$$P(q, \epsilon) = \frac{1}{2\pi(3\epsilon^N + \epsilon^C)^2} \langle \langle F_{q\sigma}^*(t); F_{q'\sigma'}(t') \rangle \rangle_{\epsilon} \quad (18)$$

where  $\epsilon^C$  is the energy of cooper pair. In Eq.(16) the delta function  $\delta_{q\sigma}$  acquires a large number of momentum and spin combinations. During the above development it is surprisingly found that the cooper pair energy  $\epsilon^C$  as well as the normal electron energy  $\epsilon^N$  automatically emerges out in the results. The solution of function  $P(q, \epsilon)$  can be obtained after decoupling of the developed Green's functions from the Green's function  $\langle \langle F_{q\sigma}^*(t); F_{q'\sigma'}(t') \rangle \rangle$ , with the help of electron and phonon renormalized Hamiltonians

$$H_{ren(e)}^{(0)} = \sum_q (\tilde{\epsilon}_{q\uparrow} b_{q\uparrow}^* b_{q\uparrow} + \tilde{\epsilon}_{q\downarrow} b_{q\downarrow}^* b_{q\downarrow} + \tilde{\epsilon}_{-q\uparrow} b_{-q\uparrow}^* b_{-q\uparrow} + \tilde{\epsilon}_{-q\downarrow} b_{-q\downarrow}^* b_{-q\downarrow}) \quad (19)$$

$$H_{ren(p)}^{(0)} = \frac{1}{4} \sum_k \left[ \frac{\tilde{\epsilon}_k^2}{\epsilon_k} A_k^* A_k + \epsilon_k B_k^* B_k \right] \quad (20)$$

As

$$\begin{aligned} P(q, \epsilon) = & \sum_{k, k'} G_{k, k'}^2 \left\{ \left[ \left( -\frac{8\tilde{\epsilon}_k^2}{\epsilon_k} + \frac{2\epsilon_k^3}{(3\epsilon^N + \epsilon^C)^2} \right) \frac{N_{Q\sigma}}{\epsilon^2 - \tilde{\epsilon}_k^2} + \left( \frac{\epsilon_k^2 n_k}{(3\epsilon^N + \epsilon^C)^2} + \frac{4\epsilon_k \tilde{n}_k}{3\epsilon^N + \epsilon^C} + \tilde{n}_k \right) \right] \right. \\ & \times \frac{4}{\epsilon - (3\tilde{\epsilon}^N + \tilde{\epsilon}^C)} + 128 \sum_{k_1} \left[ D(k_1, -k) D(-k_1, -k') \frac{\epsilon_{k_1} N_{Q\sigma}}{\epsilon^2 - \tilde{\epsilon}_{k_1}^2} + D(k_1, -k) D(k_1, -k') \right. \\ & \times \left. \frac{n_{k_1}}{\epsilon - (3\tilde{\epsilon}^N + \tilde{\epsilon}^C)} \right] \frac{1}{(3\epsilon^N + \epsilon^C)^2} + 288 \sum_{k_1, k_2} \left[ V_3(k_1, k_2, -k) V_3(-k_1, -k_2, -k') \left( \frac{S_{+\alpha} \tilde{\epsilon}_{+\alpha}}{\epsilon^2 - \tilde{\epsilon}_{+\alpha}^2} \right. \right. \\ & \left. \left. + \frac{S_{-\alpha} \tilde{\epsilon}_{-\alpha}}{\epsilon^2 - \tilde{\epsilon}_{-\alpha}^2} \right) \eta_1 N_{Q\sigma} + V_3(k_1, k_2, -k) V_3(k_1, k_2, -k') \left( \frac{n_{k_1} n_{k_2}}{\epsilon - (3\tilde{\epsilon}^N + \tilde{\epsilon}^C)} \right) \right] \frac{1}{(3\epsilon^N + \epsilon^C)^2} \\ & + 1536 \sum_{k_1, k_2, k_3} \left[ V_4(k_1, k_2, k_3, -k) V_4(-k_1, -k_2, -k_3, -k') \left( \frac{S_{+\beta} \tilde{\epsilon}_{+\beta}}{\epsilon^2 - \tilde{\epsilon}_{+\beta}^2} + 3 \frac{S_{-\beta} \tilde{\epsilon}_{-\beta}}{\epsilon^2 - \tilde{\epsilon}_{-\beta}^2} \right) \eta_2 N_{Q\sigma} \right. \\ & \left. \left. + V_4(k_1, k_2, k_3, -k) V_4(k_1, k_2, k_3, -k') \left( \frac{3n_{k_1} n_{k_2} n_{k_3}}{\epsilon - (3\tilde{\epsilon}^N + \tilde{\epsilon}^C)} \right) \right] \frac{1}{(3\epsilon^N + \epsilon^C)^2} \right\} \quad (21) \end{aligned}$$

where

$$n_k = \langle A_k A_k \rangle; \tilde{n}_k = \langle A_k B_k \rangle; \tilde{\tilde{n}}_k = \langle B_k B_k \rangle \quad (22a)$$

$$G_{k, k'}^2 = (g_k + g_k^*)(g_{k'} + g_{k'}^*) \quad (22b)$$

$$\tilde{\epsilon}_{\pm\alpha} = \tilde{\epsilon}_{k_1} \pm \tilde{\epsilon}_{k_2}; \tilde{\epsilon}_{\pm\beta} = \tilde{\epsilon}_{k_1} \pm \tilde{\epsilon}_{k_2} \pm \tilde{\epsilon}_{k_3}; \quad (22c)$$

$$S_{\pm\alpha} = n_{k_2} \pm n_{k_1}; S_{\pm\beta} = 1 \pm n_{k_1} n_{k_2} + n_{k_2} n_{k_3} \pm n_{k_3} n_{k_1} \quad (22d)$$

$$\eta_1 = \frac{\epsilon_{k_1} \epsilon_{k_2}}{\tilde{\epsilon}_{k_1} \tilde{\epsilon}_{k_2}}; \eta_2 = \frac{\epsilon_{k_1} \epsilon_{k_2} \epsilon_{k_3}}{\tilde{\epsilon}_{k_1} \tilde{\epsilon}_{k_2} \tilde{\epsilon}_{k_3}} \quad (22e)$$

and

$$N_{Q\sigma} = \int_{-\infty}^{+\infty} \frac{e^{-i\epsilon(t-t')}}{e^{\beta\epsilon} \pm 1} \delta[\epsilon - (3\tilde{\epsilon}^N + \tilde{\epsilon}^C)] d\epsilon \quad (22f)$$

The value of  $N_{Q\sigma}$  is including through the cooper pairs and normal electron problem via  $\delta[\epsilon - (3\tilde{\epsilon}^N + \tilde{\epsilon}^C)]$  function. This cannot be normally solved with usual process. Hence, here we can write above expressions as (to the reasonable degree of accuracy and without violating physical laws)

$$N_{Q\sigma} \cong \int_{-\infty}^{+\infty} e^{-i\epsilon(t-t')} \frac{1}{2} \left[ \frac{\delta(\epsilon - 3\tilde{\epsilon}^N)}{e^{\beta\epsilon} + 1} + \frac{\delta(\epsilon - \tilde{\epsilon}^C)}{e^{\beta\epsilon} - 1} \right] d\epsilon \quad (23)$$

or 
$$N_{Q\sigma} \cong \frac{1}{2} \left[ \frac{1}{e^{3\beta\tilde{\epsilon}^N} + 1} + \frac{1}{e^{\beta\tilde{\epsilon}^C} - 1} \right] \quad (t = t') \quad (24)$$

The shift  $\Delta_{(ep)}(q, \epsilon)$  and line width  $\Gamma_{(ep)}(q, \epsilon)$  can be separated in three terms, namely; (i) electron-phonon contribution, (ii) defect contribution and (iii) anharmonic contribution. Thus

$$\Delta_{(ep)}(q, \epsilon) = \Delta_{(ep)}^{EP}(q, \epsilon) + \Delta_{(ep)}^D(q, \epsilon) + \Delta_{(ep)}^A(q, \epsilon) \quad (25)$$

And

$$\Gamma_{(ep)}(q, \epsilon) = \Gamma_{(ep)}^{EP}(q, \epsilon) + \Gamma_{(ep)}^D(q, \epsilon) + \Gamma_{(ep)}^A(q, \epsilon) \quad (26)$$

With

$$\Gamma_{(ep)}^{EP}(q, \epsilon) = \Omega_v G_k^2 \left\{ \left[ -4\tilde{\epsilon}_k^2 + \frac{\tilde{\epsilon}_k^4}{(3\tilde{\epsilon}^N + \tilde{\epsilon}^C)^2} \right] N_{Q\sigma} + 2 \left[ \frac{\epsilon_k^2 n_k}{(3\tilde{\epsilon}^N + \tilde{\epsilon}^C)^2} + \frac{4\epsilon_k \tilde{n}_k}{(3\tilde{\epsilon}^N + \tilde{\epsilon}^C)} + 4\tilde{n}_k \right] (3\tilde{\epsilon}^N + \tilde{\epsilon}^C)^2 \right\} \quad (27a)$$

$$\Gamma_{(ep)}^D(q, \epsilon) = 32\Omega_v \sum_{k_1} |D(k_1, -k)|^2 G_k^2 [\epsilon_{k_1} \tilde{\epsilon}_{k_1} N_{Q\sigma} + n_{k_1} (3\tilde{\epsilon}^N + \tilde{\epsilon}^C)^2] \frac{1}{(3\tilde{\epsilon}^N + \tilde{\epsilon}^C)^2} \quad (27b)$$

$$\Gamma_{(ep)}^{3A}(q, \epsilon) = 72\Omega_v \sum_{k_1, k_2} G_k^2 |V_3(k_1, k_2, -k)|^2 \eta_1 \{ [S_{+\alpha} \tilde{\epsilon}_{+\alpha} + S_{-\alpha} \tilde{\epsilon}_{-\alpha}] N_{Q\sigma} + (3\tilde{\epsilon}^N + \tilde{\epsilon}^C)^2 n_{k_1} n_{k_2} \} \frac{1}{(3\tilde{\epsilon}^N + \tilde{\epsilon}^C)^2} \quad (27c)$$

$$\Gamma_{(ep)}^{4A}(q, \epsilon) = 384\Omega_v \sum_{k_1, k_2, k_3} G_k^2 |V_3(k_1, k_2, k_3, -k)|^2 \{ \eta_2 [S_{+\beta} \tilde{\epsilon}_{+\beta} + 3S_{-\beta} \tilde{\epsilon}_{-\beta}] N_{Q\sigma} + (3\tilde{\epsilon}^N + \tilde{\epsilon}^C)^2 n_{k_1} n_{k_2} n_{k_3} \} \frac{1}{(3\tilde{\epsilon}^N + \tilde{\epsilon}^C)^2} \quad (27d)$$

## 5 Electronic Raman Scattering

The theory of the electronic Raman scattering in superconductors was pioneered by Abrikosov and Fal'kovskii [18], Abrikosov and Genkin [19], and together with experiments on  $V_3Si$ , a superconductor with a  $T_c$  of 17K. The contributions of electrons to the Raman signal gives information about the excitations at the Fermi surface and have about the pairing mechanism, which is difficult to obtain otherwise. As it is already calculated the electron-phonon energy line width  $\Gamma_{(ep)}(q, \epsilon)$  in Eq.(25), but here it is being adopted the same methodology to calculate the electron-phonon energy line width with addition of polarization index  $j$  in the Hamiltonian (14). For electronic Raman scattering the spectral density function can easily be obtained on substitution of imaginary part of electron Green's function given by (27) into (11)

$$i_{\alpha\gamma, \beta\lambda}^{(ep)}(kj, q, \epsilon) = \left( \frac{1}{2\pi^2} \right) \sum_{k_1, j_1, k_1', j_1'} \int_{-\infty}^{+\infty} dt \exp(-i\epsilon t) P_{\alpha\gamma, \beta\lambda}^{(1)} \left( \begin{matrix} k_1 & k_1' \\ j_1 & j_1' \end{matrix} \right) \int_{-\infty}^{+\infty} d\epsilon' (3\tilde{\epsilon}^N + \tilde{\epsilon}^C)^2 \times \left( \frac{\exp(i\epsilon t)}{[\exp(\beta\epsilon) \pm 1]} \right) \left\{ \frac{\Gamma_{(ep)}(kj, q, \epsilon)}{[(\epsilon^2 - \bar{\epsilon}_q^2)^2 + (3\tilde{\epsilon}^N + \tilde{\epsilon}^C)^2 \Gamma_{(ep)}^2(kj, q, \epsilon)]^2} \right\} \quad (28)$$

The above expression follows for bosons and fermions, therefore, the  $\pm$  sign defined as; the  $+$  sign for fermions and  $-$  sign for bosons. After evaluating Eq.(28) the electronic Raman scattering can be written as

$$i_{\alpha\gamma,\beta\lambda}^{(ep)}(kjq, \epsilon) = \left(\frac{1}{2\pi^2}\right) \sum_{k_1 j_1, k_1' j_1'} P_{\alpha\gamma,\beta\lambda}^{(1)} \left(\begin{matrix} k_1 & k_1' \\ j_1 & j_1' \end{matrix}\right) \int_{-\infty}^{+\infty} d\epsilon \in (3\epsilon^N + \epsilon^C)^2 \left[ \frac{1}{\exp(\beta\epsilon) \pm 1} \right] \frac{\Gamma_{(ep)}(kjq, \epsilon)}{[\epsilon^2 - \bar{\epsilon}_q^2]^2} \quad (29)$$

### 5.1 First-order Electronic Raman Scattering

The first order Raman scattering can be obtained in this form

$$i_{\alpha\gamma,\beta\lambda}^{(ep)}(kjq, \epsilon)_{\text{FOR}} = \left(\frac{32}{\pi}\right) \sum_{k_j, k_j', k_1 j_1} P_{\alpha\gamma,\beta\lambda}^{(1)} \left(\begin{matrix} k_1-k \\ j_1 \end{matrix}\right) G^2(k) \left\{ D_{j_1 j}^{(k_1-k)} D_{j_1 j'}^{(-k_1-k')} [n(\epsilon^C) + n(\epsilon^N)] \right. \\ \left. \times \frac{n[\tilde{\epsilon} \left(\begin{matrix} k_1 \\ j_1 \end{matrix}\right)] \epsilon \left(\begin{matrix} k_1 \\ j_1 \end{matrix}\right)}{\tilde{\epsilon} \left(\begin{matrix} k_1 \\ j_1 \end{matrix}\right) [\tilde{\epsilon}^2 \left(\begin{matrix} k_1 \\ j_1 \end{matrix}\right) - \bar{\epsilon}_q^2]^2} + D_{j_1 j}^{(k_1-k)} D_{j_1 j'}^{(k_1-k')} \frac{n \left(\begin{matrix} k_1 \\ j_1 \end{matrix}\right) [n(\tilde{\epsilon}^C) + n(\tilde{\epsilon}^N)]}{[(3\tilde{\epsilon}^N + \tilde{\epsilon}^C)^2 - \bar{\epsilon}_q^2]^2} \right\} \quad (30)$$

where

$$n(\epsilon^N) = \frac{1}{[\exp(3\beta\epsilon^N) + 1]} ; \quad n(\epsilon^C) = \frac{1}{[\exp(\beta\epsilon^C) - 1]} \quad (31)$$

The first-order Raman scattering is depends on the force constant change parameter. In obtaining Eq.(30), only the electron-phonon line width  $\Gamma_{(ep)}^D(kjq, \epsilon)$  generated by the localized interactions is used. The FOR scattering is found dependent of temperature via terms  $n\left\{\tilde{\epsilon} \left(\begin{matrix} k_1 \\ j_1 \end{matrix}\right)\right\}$ ,  $n \left(\begin{matrix} k_1 \\ j_1 \end{matrix}\right)$ ,  $n(\epsilon^C)$  and  $n(\epsilon^N)$ . After substitution of Eq.(28) into Eq.(12) with appropriate simplifications, one can very easily obtain the FOR scattering differential cross section in the form

$$\left(\frac{d^2\sigma}{d\Omega d\epsilon_R}\right)_{\text{FOR}} = \left(\frac{\epsilon}{c}\right)^4 \sum_{\alpha\beta,\gamma\lambda} m_\gamma m_\beta n_\alpha n_\beta i_{\alpha\gamma,\beta\lambda}^{(ep)}(kjq, \epsilon)_{\text{FOR}} \quad (32)$$

### 5.2 First-order electronic Raman scattering for high-T<sub>c</sub> superconductor

The first-order electronic Raman scattering for high temperature superconductors can be examined in two ways: (i) in the close vicinity of T<sub>c</sub> the superconductivity increases and (ii) the contribution of normal energy becomes small compared to the cooper pair energy.

**Case I** ( $\epsilon^N \rightarrow 0$ ):

$$i_{\alpha\gamma,\beta\lambda}^{(ep)}(kj, C)_{\text{FOR}} = \left(\frac{16}{\pi}\right) \sum_{k_j, k_j', k_1 j_1} P_{\alpha\gamma,\beta\lambda}^{(1)} \left(\begin{matrix} k_1-k \\ j_1 \end{matrix}\right) G^2(k) \left\{ D_{j_1 j}^{(k_1-k)} D_{j_1 j'}^{(-k_1-k')} n(\epsilon^C) n\left[\tilde{\epsilon} \left(\begin{matrix} k_1 \\ j_1 \end{matrix}\right)\right] \frac{\epsilon \left(\begin{matrix} k_1 \\ j_1 \end{matrix}\right)}{\tilde{\epsilon} \left(\begin{matrix} k_1 \\ j_1 \end{matrix}\right)} \right. \\ \left. \times \left[ \frac{1}{(\epsilon^C)^4} - \frac{2\tilde{\epsilon}^2 \left(\begin{matrix} k_1, C \\ j_1 \end{matrix}\right) - \tilde{\epsilon}^2 \left(\begin{matrix} k_1 \\ j_1 \end{matrix}\right)}{(\epsilon^C)^6} \right] + D_{j_1 j}^{(k_1-k)} D_{j_1 j'}^{(k_1-k')} n \left(\begin{matrix} k_1 \\ j_1 \end{matrix}\right) \right. \\ \left. \times \left[ \frac{1}{(\epsilon^C)^4} - \frac{2\tilde{\epsilon}^2 \left(\begin{matrix} k_1, C \\ j_1 \end{matrix}\right) - \tilde{\epsilon}^2 \left(\begin{matrix} k_1 \\ j_1 \end{matrix}\right)}{(\epsilon^C)^6} \right] n(\tilde{\epsilon}^C) \right\} \quad (33)$$



**Case II** ( $\epsilon^C \gg \epsilon^N$ ):

$$i_{\alpha\gamma,\beta\lambda}^{(ep)}(kj, q)_{\text{FOR}} = \left(\frac{16}{\pi}\right) \sum_{kj, kj', k_1 j_1} P_{\alpha\gamma,\beta\lambda}^{(1)} \left(\begin{matrix} k_1-k \\ j_1 \ j \end{matrix}\right) G^2(k) \left\{ D\left(\begin{matrix} k_1-k \\ j_1 \ j \end{matrix}\right) D\left(\begin{matrix} -k_1-k' \\ j_1 \ j' \end{matrix}\right) n(\epsilon^C) n[\tilde{\epsilon} \left(\begin{matrix} k_1 \\ j_1 \end{matrix}\right)] \frac{\epsilon \left(\begin{matrix} k_1 \\ j_1 \end{matrix}\right)}{\tilde{\epsilon} \left(\begin{matrix} k_1 \\ j_1 \end{matrix}\right)} \right. \\ \times \left[ \frac{1}{(\epsilon^C)^4} - \frac{2\tilde{\epsilon}^2 \left(\begin{matrix} k_1 \\ j_1 \end{matrix}\right), C}{(\epsilon^C)^6} - \tilde{\epsilon}^2 \left(\begin{matrix} k_1 \\ j_1 \end{matrix}\right) \right] + D\left(\begin{matrix} k_1-k \\ j_1 \ j \end{matrix}\right) D\left(\begin{matrix} k_1-k' \\ j_1 \ j' \end{matrix}\right) \left( \frac{1}{(\tilde{\epsilon}^C)^2 (\tilde{\epsilon}^C + \epsilon^C)^2} \right) n\left(\begin{matrix} k_1 \\ j_1 \end{matrix}\right) \\ \times \left( 1 - \frac{6(\tilde{\epsilon}^N + \epsilon^N)}{(\tilde{\epsilon}^C + \epsilon^C)} \right) \left[ 1 - \frac{2\tilde{\epsilon}^2 \left(\begin{matrix} k_1 \\ j_1 \end{matrix}\right), C}{(\tilde{\epsilon}^C + 3\tilde{\epsilon}^N)^2 - (\epsilon^C + 3\epsilon^N)^2} \right] \left( 1 + \frac{2\{\epsilon^C - (\tilde{\epsilon}^N + \epsilon^N)\}}{\tilde{\epsilon}^C} \right) n(\tilde{\epsilon}^C) \left. \right\} \quad (34)$$

where

$$\tilde{\epsilon}^2(k, C) \cong (\epsilon^C) \Delta(k, C) - 12 \sum_{k, k_1} (g_k + g_k^*) \mathcal{V}_3(k_1, k_1, -k) n_{k_1} \quad (35)$$

### 5.3 Higher-order Electronic Raman Scattering

#### 5.3.1 Second-order electronic Raman scattering

The second-order electronic Raman tensor is given by

$$i_{\alpha\gamma,\beta\lambda}^{(ep)}(kjq, \epsilon)_{\text{SOR}} = i_{\alpha\gamma,\beta\lambda}^{(ep),1}(kjq, \epsilon)_{\text{SOR}} + i_{\alpha\gamma,\beta\lambda}^{(ep),2}(kjq, \epsilon)_{\text{SOR}} \quad (36)$$

The first term is obtained, for this processes in which two phonons contribute in inelastic scattering processes via  $\Gamma_{(ep)}^{3A}(kjq, \epsilon)$  and is given by

$$i_{\alpha\gamma,\beta\lambda}^{(ep),1}(kjq, \epsilon)_{\text{SOR}} = \left(\frac{72}{\pi}\right) \sum_{kj, kj', k_1 j_1, k_2 j_2} P_{\alpha\gamma,\beta\lambda}^{(1)} \left(\begin{matrix} k_1-k \\ j_1 \ j \end{matrix}\right) G^2(k) \left\{ V_3\left(\begin{matrix} k_1 k_2-k \\ j_1 j_2 \ j \end{matrix}\right) V_3\left(\begin{matrix} -k_1-k_2-k' \\ j_1 \ j_2 \ j' \end{matrix}\right) \eta_1 [n(\epsilon^C) + n(\epsilon^N)] \tilde{n}(\tilde{\epsilon}_{\pm\alpha}) S_1^{(\pm)} \right. \\ \left. + V_3\left(\begin{matrix} k_1 k_2-k \\ j_1 j_2 \ j \end{matrix}\right) V_3\left(\begin{matrix} k_1 k_2-k' \\ j_1 j_2 \ j' \end{matrix}\right) n\left(\begin{matrix} k_1 \\ j_1 \end{matrix}\right) \frac{n\left(\begin{matrix} k_2 \\ j_2 \end{matrix}\right)}{[(3\tilde{\epsilon}^N + \tilde{\epsilon}^C)^2 - \tilde{\epsilon}^2 \left(\begin{matrix} k \\ j \end{matrix}\right)]^2} [n(\tilde{\epsilon}^C) + n(\tilde{\epsilon}^N)] \right\} \quad (37)$$

and second term  $i_{\alpha\gamma,\beta\lambda}^{(ep),2}(kjq, \epsilon)_{\text{SOR}}$  emerges due to the fluctuations in the second-order electronic polarizability and can be obtained as

$$i_{\alpha\gamma,\beta\lambda}^{(ep),2}(kjq, \epsilon)_{\text{SOR}} = \left(\frac{1}{2\pi}\right) \sum_{k_1 j_1, k_2 j_2} P_{\alpha\gamma,\beta\lambda}^{(2)} \left(\begin{matrix} k_1 k_2-k_1-k_2 \\ j_1 j_2 \ j_1 \ j_2 \end{matrix}\right) \eta_1 \left[ \tilde{n}(\tilde{\epsilon}_{+\alpha}) S_1^{(+)} + \tilde{n}(\tilde{\epsilon}_{-\alpha}) S_1^{(-)} \right] \quad (38)$$

The SOR tensor consists of two terms,  $i_{\alpha\gamma,\beta\lambda}^{(ep),1}(kjq, \epsilon)_{\text{SOR}}$  and  $i_{\alpha\gamma,\beta\lambda}^{(ep),2}(kjq, \epsilon)_{\text{SOR}}$ , of which the first term come from the FOR-type excitation, while the second term appears from the second derivative of  $P_{\beta\lambda}(t)$ . This term is heavily influenced by the temperature dependent distribution function contained in  $\tilde{n}(\tilde{\epsilon}_{+\alpha}) S_1^{(+)}$ .

### 5.3.1.1 Second-order electronic Raman scattering for high- $T_c$ superconductor

The second-order electronic polarizability is not appears in the second-order electronic Raman scattering for high- $T_c$  superconductor. Here only two phonons contributed Raman tensor term is appears.

**Case I** ( $\epsilon^N \rightarrow 0$ ):

$$i_{\alpha\gamma,\beta\lambda}^{(ep),1}(kj, C)_{\text{SOR}} = \left(\frac{36}{\pi}\right) \sum_{\substack{kj, k'j' \\ k_1j_1, k_2j_2}} P_{\alpha\gamma,\beta\lambda}^{(1)}(k_1-k) G^2(k) \left\{ V_3 \left( \begin{matrix} k_1 & k_2 & -k \\ j_1 & j_2 & j \end{matrix} \right) V_3 \left( \begin{matrix} -k_1 & -k_2 & -k' \\ j_1 & j_2 & j' \end{matrix} \right) \eta_1 n(\epsilon^C) \tilde{n}(\tilde{\epsilon}_{\pm\alpha}) \left( \frac{S_1^{(\pm)}}{\tilde{\epsilon}_{\pm\alpha}} \right) \right. \\ \times \left[ \frac{1}{(\epsilon^C)^4} - \frac{2\tilde{\epsilon}^2 \left( \begin{matrix} k_1 \\ j_1 \end{matrix} \right) C - \tilde{\epsilon}_{\pm\alpha}^2}{(\epsilon^C)^6} \right] + V_3 \left( \begin{matrix} k_1 & k_2 & -k \\ j_1 & j_2 & j \end{matrix} \right) V_3 \left( \begin{matrix} k_1 & k_2 & -k' \\ j_1 & j_2 & j' \end{matrix} \right) n \left( \begin{matrix} k_1 \\ j_1 \end{matrix} \right) n \left( \begin{matrix} k_2 \\ j_2 \end{matrix} \right) \\ \left. \times \left[ \frac{1}{(\tilde{\epsilon}^C)^4} - \frac{2\tilde{\epsilon}^2 \left( \begin{matrix} k_1 \\ j_1 \end{matrix} \right) C - (\epsilon^C)^2}{(\tilde{\epsilon}^C)^6} \right] n(\tilde{\epsilon}^C) \right\} \quad (39)$$

**Case II** ( $\epsilon^C \gg \epsilon^N$ ):

$$i_{\alpha\gamma,\beta\lambda}^{(ep),1}(kj, q)_{\text{SOR}} = \left(\frac{36}{\pi}\right) \sum_{\substack{kj, k'j' \\ k_1j_1, k_2j_2}} P_{\alpha\gamma,\beta\lambda}^{(1)}(k_1-k) G^2(k) \left\{ V_3 \left( \begin{matrix} k_1 & k_2 & -k \\ j_1 & j_2 & j \end{matrix} \right) V_3 \left( \begin{matrix} -k_1 & -k_2 & -k' \\ j_1 & j_2 & j' \end{matrix} \right) \eta_1 n(\epsilon^C) \tilde{n}(\tilde{\epsilon}_{\pm\alpha}) \left( \frac{S_1^{(\pm)}}{\tilde{\epsilon}_{\pm\alpha}} \right) \right. \\ \times \left[ \frac{1}{(\epsilon^C)^4} - \frac{2\tilde{\epsilon}^2 \left( \begin{matrix} k_1 \\ j_1 \end{matrix} \right) C - \tilde{\epsilon}_{\pm\alpha}^2}{(\epsilon^C)^6} \right] + V_3 \left( \begin{matrix} k_1 & k_2 & -k \\ j_1 & j_2 & j \end{matrix} \right) V_3 \left( \begin{matrix} k_1 & k_2 & -k' \\ j_1 & j_2 & j' \end{matrix} \right) n \left( \begin{matrix} k_1 \\ j_1 \end{matrix} \right) n \left( \begin{matrix} k_2 \\ j_2 \end{matrix} \right) \\ \times \left[ \frac{1}{(\tilde{\epsilon}^C)^2 (\tilde{\epsilon}^C + \epsilon^C)^2} \right] \left[ 1 - \frac{6(\tilde{\epsilon}^N + \epsilon^N)}{(\tilde{\epsilon}^C + \epsilon^C)} \right] \left[ 1 + \frac{2\{\epsilon^C - (\tilde{\epsilon}^N + \epsilon^N)\}}{\tilde{\epsilon}^C} \right] \\ \left. \times \left[ 1 - \frac{2\tilde{\epsilon}^2 \left( \begin{matrix} k_1 \\ j_1 \end{matrix} \right) C}{(\tilde{\epsilon}^C + 3\tilde{\epsilon}^N)^2 - (\epsilon^C + 3\epsilon^N)^2} \right] n(\tilde{\epsilon}^C) \right\} \quad (40)$$

The SOR cross section can be obtained in the form

$$\left( \frac{d^2\sigma}{d\Omega d\epsilon_R} \right)_{\text{SOR}} = \left( \frac{\epsilon}{c} \right)^4 \sum_{\alpha\beta,\gamma\lambda} m_\gamma m_\beta n_\alpha n_\beta \left[ i_{\alpha\gamma,\beta\lambda}^{(ep),1}(kj, \epsilon)_{\text{SOR}} + i_{\alpha\gamma,\beta\lambda}^{(ep),2}(kj, \epsilon)_{\text{SOR}} \right] \quad (41)$$

### 5.3.3 Third-order Electronic Raman Scattering

After certain simplifications the third-order electronic Raman scattering tensor takes the form

$$i_{\alpha\gamma,\beta\lambda}^{(ep)}(kj, \epsilon)_{\text{TOR}} = i_{\alpha\gamma,\beta\lambda}^{(ep),1}(kj, \epsilon)_{\text{TOR}} + i_{\alpha\gamma,\beta\lambda}^{(ep),2}(kj, \epsilon)_{\text{TOR}} \quad (42)$$

where in the first term the first-order electronic polarizability is involved and the second term appears due to the fluctuations of the electronic polarizability in its third-order derivative; these terms are given by

$$i_{\alpha\gamma,\beta\lambda}^{(ep),1}(kj, \epsilon)_{\text{TOR}} = \left(\frac{384}{\pi}\right) \sum_{\substack{kj, k'j', k_1j_1 \\ k_2j_2, k_3j_3}} P_{\alpha\gamma,\beta\lambda}^{(1)}(k_1-k) G^2(k) \left\{ V_4 \left( \begin{matrix} k_1 & k_2 & k_3 & -k \\ j_1 & j_2 & j_3 & j \end{matrix} \right) V_4 \left( \begin{matrix} -k_1 & -k_2 & -k_3 & -k' \\ j_1 & j_2 & j_3 & j' \end{matrix} \right) n(\epsilon^C) + n(\epsilon^N) \right\} S_2(\beta) \\ + 3V_4 \left( \begin{matrix} k_1 & k_2 & k_3 & -k \\ j_1 & j_2 & j_3 & j \end{matrix} \right) V_4 \left( \begin{matrix} k_1 & k_2 & k_3 & -k' \\ j_1 & j_2 & j_3 & j' \end{matrix} \right) n \left( \begin{matrix} k_1 \\ j_1 \end{matrix} \right) n \left( \begin{matrix} k_2 \\ j_2 \end{matrix} \right) n \left( \begin{matrix} k_3 \\ j_3 \end{matrix} \right) \frac{1}{(3\tilde{\epsilon}^N + \tilde{\epsilon}^C)^2 - \tilde{\epsilon}^2 \left( \begin{matrix} k \\ j \end{matrix} \right)^2} [n(\tilde{\epsilon}^C) + n(\tilde{\epsilon}^N)] \quad (42)$$

and

$$i_{\alpha\gamma,\beta\lambda}^{(ep),2}(kjq, \epsilon)_{\text{TOR}} = \left(\frac{1}{2\pi}\right) \sum_{\substack{k_1 j_1, \\ k_2 j_2, k_3 j_3}} P_{\alpha\gamma,\beta\lambda}^{(3)} \binom{k_1 k_2 k_3 - k_1 - k_2 - k_3}{j_1 j_2 j_3} n_2 [\tilde{n}_1 S_1 + \tilde{n}_2 S_2 + \tilde{n}_3 S_3 + \tilde{n}_4 S_4] \quad (43)$$

$$S \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} = 1 \begin{pmatrix} + \\ - \\ - \\ + \end{pmatrix} n \binom{k_1}{j_1} n \binom{k_2}{j_2} \begin{pmatrix} + \\ + \\ - \\ - \end{pmatrix} n \binom{k_2}{j_2} n \binom{k_3}{j_3} \begin{pmatrix} + \\ - \\ - \\ + \end{pmatrix} n \binom{k_3}{j_3} n \binom{k_1}{j_1} \quad (4.32c)$$

$$\epsilon \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} + \\ - \\ + \\ + \end{pmatrix} \cong \binom{k_1}{j_1} \begin{pmatrix} + \\ + \\ - \\ - \end{pmatrix} \cong \binom{k_2}{j_2} \begin{pmatrix} + \\ + \\ - \\ - \end{pmatrix} \cong \binom{k_3}{j_3} \quad (4.32d)$$

$$\tilde{n} \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} = \left\{ \exp \left[ \beta \hbar \epsilon \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} \right] - 1 \right\}^{-1} \quad (4.32e)$$

The third-order Raman scattering is induced by three types of terms, two of which, namely diagonal and non-diagonal appear due to processes in which light suffers three successive first-order Raman scattering and third term appears due to the processes in which the third-order derivative of the electronic polarizability is encountered. It is notable that no second-order derivative is involved in third-order Raman scattering [19]. The intensity of TOR lines is also affected by the temperature. The various TOR lines with different temperature functions may be obtained at the energies

$$\begin{aligned} & \pm 3 \cong \binom{k_1}{j_1}, \pm 3 \cong \binom{k_1}{j_2}, \pm 3 \cong \binom{k_3}{j_3}, \pm \cong \binom{-k_1}{j_2}, \pm \cong \binom{-k_1}{j_3} \\ & \pm \left[ \begin{pmatrix} + \\ - \\ + \\ - \end{pmatrix} \cong \binom{k_1}{j_1} \begin{pmatrix} + \\ + \\ - \\ + \end{pmatrix} \cong \binom{k_2}{j_2} \begin{pmatrix} + \\ + \\ - \\ - \end{pmatrix} \cong \binom{k_3}{j_3} \right] \pm \left[ \begin{pmatrix} + \\ - \\ + \\ + \end{pmatrix} \cong \binom{k_1}{j_1} \begin{pmatrix} + \\ + \\ - \\ - \end{pmatrix} \cong \binom{-k_1}{j_2} \begin{pmatrix} + \\ + \\ - \\ - \end{pmatrix} \cong \binom{-k_1}{j_1} \right] \\ & \pm \left[ \begin{pmatrix} + \\ - \\ + \\ + \end{pmatrix} \cong \binom{k_1}{j_1} \begin{pmatrix} + \\ + \\ - \\ + \end{pmatrix} \cong \binom{-k_1}{j_2} \begin{pmatrix} + \\ + \\ - \\ - \end{pmatrix} \cong \binom{-k_1}{j_2} \right] \end{aligned}$$

The third-order electronic Raman scattering cross section can also be obtained in the form

$$\left(\frac{d^2\sigma}{d\Omega d\epsilon_R}\right)_{\text{TOR}} = \left(\frac{\epsilon}{c}\right)^4 \sum_{\alpha\beta,\gamma\lambda} m_\gamma m_\beta n_\alpha n_\beta \left[ i_{\alpha\gamma,\beta\lambda}^{(ep),1}(kjq, \epsilon)_{\text{TOR}} + i_{\alpha\gamma,\beta\lambda}^{(ep),2}(kjq, \epsilon)_{\text{TOR}} \right] \quad (44)$$

**5.3.3.1 Third-order electronic Raman scattering for high- $T_c$  superconductor**

**Case I** ( $\epsilon^N \rightarrow 0$ ):

$$i_{\alpha\gamma,\beta\lambda}^{(ep),1}(kj,C)_{\text{TOR}} = \left(\frac{192}{\pi}\right) \sum_{\substack{kj,kj',k_1j_1 \\ k_2j_2,k_3j_3}} P_{\alpha\gamma,\beta\lambda}^{(1)} \binom{k_1-k}{j_1 j} G^2(k) \left\{ V_4 \binom{k_1 k_2 k_3-k}{j_1 j_2 j_3 j} V_4 \binom{-k_1-k_2-k_3-k'}{j_1 j_2 j_3 j'} \right\} n_2 n(\epsilon^C) \tilde{S}_2(\beta) \\ + 3V_4 \binom{k_1 k_2 k_3-k}{j_1 j_2 j_3 j} V_4 \binom{k_1 k_2 k_3-k'}{j_1 j_2 j_3 j'} n \binom{k_1}{j_1} n \binom{k_2}{j_2} n \binom{k_3}{j_3} \left[ \frac{1}{(\tilde{\epsilon}^C)^4} - \frac{2\tilde{\epsilon}^2 \binom{k_1}{j_1}, C) - (\epsilon^C)^2}{(\tilde{\epsilon}^C)^6} \right] n(\tilde{\epsilon}^C) \} \quad (45)$$

where

$$\tilde{S}_2(\beta) = \sum_{i=1}^4 \tilde{n}_i S_i \tilde{\epsilon}_i^{-1} \left\{ (\epsilon^C)^{-4} - [2\tilde{\epsilon}^2 \binom{k_1}{j_1}, C) - \tilde{\epsilon}_i^2] (\epsilon^C)^{-6} \right\} \quad (46)$$

**Case II** ( $\epsilon^C \gg \epsilon^N$ ):

$$i_{\alpha\gamma,\beta\lambda}^{(ep),1}(kj,q)_{\text{TOR}} = \left(\frac{192}{\pi}\right) \sum_{\substack{kj,kj',k_1j_1 \\ k_2j_2,k_3j_3}} P_{\alpha\gamma,\beta\lambda}^{(1)} \binom{k_1-k}{j_1 j} G^2(k) \left\{ V_4 \binom{k_1 k_2 k_3-k}{j_1 j_2 j_3 j} V_4 \binom{-k_1-k_2-k_3-k'}{j_1 j_2 j_3 j'} \right\} n_2 n(\epsilon^C) \tilde{S}_2(\beta) \\ + 3V_4 \binom{k_1 k_2 k_3-k}{j_1 j_2 j_3 j} V_4 \binom{k_1 k_2 k_3-k'}{j_1 j_2 j_3 j'} n \binom{k_1}{j_1} n \binom{k_2}{j_2} n \binom{k_3}{j_3} \left[ \frac{1}{(\tilde{\epsilon}^C)^2 (\tilde{\epsilon}^C + \epsilon^C)^2} \right] n(\tilde{\epsilon}^C) \} \\ \times \left[ 1 + \frac{2\{\epsilon^C - (\tilde{\epsilon}^N + \epsilon^N)\}}{\tilde{\epsilon}^C} \right] \left[ 1 - \frac{6(\tilde{\epsilon}^N + \epsilon^N)}{(\tilde{\epsilon}^C + \epsilon^C)} \right] \left[ 1 - \frac{2\tilde{\epsilon}^2 \binom{k_1}{j_1}, C)}{(\tilde{\epsilon}^C + 3\tilde{\epsilon}^N)^2 - (\epsilon^C + 3\epsilon^N)^2} \right] \quad (47)$$

The electronic Raman scattering for high temperature superconductors will be difficult task to explain with numerical calculations, because the term  $\tilde{\epsilon}^2 \binom{k_1}{j_1}, C)$  includes with shift energy.

## 6 CONCLUSION

It emerges from the present study that electron Raman scattering in High temperature superconductors can be successfully explained with the help of present (defect induced anharmonic crystal) theory. It is observed that the FOR scattering provides only one Stokes component and one anti-Stokes component and is induced due to processes in which fluctuations in the first order electronic polarizability are encountered. This scattering is governed by the defect terms only. The SORS is contributed due to three terms, namely: diagonal and non-diagonal terms appearing due to the fluctuations in the first-order electronic polarizability and third term emerges from Eq.(6b) due to the fluctuations in the second-order electronic polarizability. The electron-phonon Raman scattering can be obtained via using the electron-phonon contribution in density of states and separated into two classes. These classes are also temperature dependent via  $\tilde{n}(2\tilde{\epsilon}_Q)$  and  $N_Q$ .

We have also investigated the electronic Raman scattering in three orders namely; FORS, SORS and TORS. The first-order Raman scattering is depends on the force constant change parameter which, obtained via electron-phonon line width  $\Gamma_{(ep)}^D(kjq, \epsilon)$ . The FOR scattering is found dependent of temperature via terms  $n\left\{\tilde{\epsilon}\left(\begin{smallmatrix} k_1 \\ j_1 \end{smallmatrix}\right)\right\}, n\left(\begin{smallmatrix} k_1 \\ j_1 \end{smallmatrix}\right), n(\epsilon^C)$  and  $n(\epsilon^N)$ . The first-order electronic Raman scattering for high temperature superconductors can be examined in two ways: (i) in the close vicinity of Tc the superconductivity increases and (ii) the contribution of normal energy becomes small compared to the cooper pair energy.

The second order electronic Raman scattering is governed by two types of terms; the first category of terms contributes due to the fluctuations in the first-order electronic polarizability influenced by the cubic anharmonicity. The second type of category of second order electronic Raman scattering is appearing due to the fluctuations in the second-order electronic polarizability. This term is heavily influenced by the temperature dependent distribution function contained in  $\tilde{n}(\tilde{\epsilon}_{+\alpha})S_1^{(+)}$ . The second-order electronic polarizability is not appears in the second-order electronic Raman scattering for high Tc superconductor. Here only two phonons contributed Raman tensor term is appears. Third-order electronic Raman scattering also obtained into two terms; in the first term the first-order electronic polarizability is involved and the second term appears due to the fluctuations of the electronic polarizability in its third-order derivative.

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