BIFURCATION AND CHAOS ON PHONEMES OF SOME LANGUAGES IN NORTH EAST INDIA

Rashmi Dutta
Associate Professor
Department of Physics
North Gauhati College, Guwahati 781031
Assam State, India
Email: rashmidutta_1956@yahoo.com

Abstract: Bifurcations and Chaos is an emerging research area in theoretical Physics and many sophisticated techniques in Physics have been developed in order to carry out significant study in Language Technology. This paper deals with a comprehensive study of the occurrences of the bifurcation and chaos [2,6,7,12] in speech spectra [9]. The bifurcation and chaos throughout the spectral characteristics of three link languages of North-East India, namely, Assamese, Bodo and Rabha vowels is illustrated with suitable examples, and finally some specific conclusions are drawn.

Key Words: Bifurcation/Chaos/ Phonemes/ Language Technology
1. Introduction: At first, some useful concepts which are necessary for our purpose are explained below:

(i) Chaos: Generally, there is no accepted definition of Chaos. From a practical point of view, chaos can be defined as a bounded steady state behavior that is not an equilibrium point, not periodic, and not quasi-periodic. A noise like spectrum is a characteristic of chaotic systems. Another important fact about the chaotic systems is that the limit set for chaotic behavior is not a simple geometrical object like a circle or a torus, but is related to fractals and Cantor sets. In short, chaos can be defined as effectively unpredictable long time behavior arising in a deterministic dynamical system because of sensitivity to the initial conditions.

(ii) Stable and Unstable Periodic Points: Let \( f : \mathbb{R}^n \to \mathbb{R}^n \) be a diffeomorphism. A point \( x \) in \( \mathbb{R}^n \) is called a fixed point of \( f \) if \( f(x) = x \). A fixed point \( x \) is said to be stable if for every neighbourhood \( U \) of \( x \), there exists a neighbourhood \( V \) of \( x \) whose images \( f(V) \) lie in \( U \) for all positive integers \( k \). Otherwise, \( x \) is known to be unstable. A periodic orbit of \( f \) is a finite sequence of distinct points each of which is the image of the previous one and whose first point is the image of the last. Its period \( k \) is the number of points in the sequence, which are called periodic points of period \( k \). A periodic orbit of period \( k \) (or \( k \)-cycle) is said to be stable or unstable according as each of its points is stable or unstable when considered as a fixed point of \( f^{(k)} \), \( (f^{(k)} \) means \( k \)-times iteration of the map \( f \)).

(iii) Bifurcations: A bifurcation is a qualitative change in dynamics upon a small variation in the parameters of a system. Many dynamical systems depend on parameters. Normally, a gradual variation of a parameter in the system corresponds to the bifurcations. However, there exists a large number of problems for which the number of solutions changes abruptly and the structure of solution manifolds varies dramatically when a parameter passes through some critical values. This kind of phenomenon is called bifurcation and these parameter values are called bifurcation values (or bifurcation points).

(iv) Routes to Chaos: Chaos theory began at the end of nineteenth century (around 1890) with some great initial ideas, concepts and results of the famous French mathematician, Henri Poincare. Also the more recent path of the theory has many fascinating successful stories. Probably, the most beautiful and important one is the route from order into chaos, i.e. the Feigenbaum universality. Mitchell J. Feigenbaum, a renowned American expert in Physics, is known as the founder of the period-doubling bifurcation that may be described as a universal route to chaos – an exciting discovery in nonlinear dynamical systems. Many new universal properties have been discovered by Feigenbaum for families of maps which depend on a parameter \( \lambda \). One of his fascinating discoveries is that if a family “\( f \)” represents period-doubling bifurcation then there is an infinite sequence \( \{ \lambda_n \}_{n=1}^{\infty} \) of bifurcation values such that

\[
\lim_{n \to \infty} \frac{\lambda_n - \lambda_{n-1}}{\lambda_{n+1} - \lambda_n} = \delta
\]

where \( \delta \) is a universal number known as the Feigenbaum constant, which does not depend at all on the form of the specific family of maps. The value of \( \delta \) is 4.6692016091029........... in the dissipative case and 8.72109720.... in
the conservative case. Furthermore, his observation suggested that there is a universal size scaling in the period-doubling sequence designated as the Feigenbaum $\alpha$-value:

$$\alpha = \lim_{n \to \infty} \frac{d_n}{d_{n+1}} = 2.50290787 \ldots$$

where $d_n$ is the ‘size’ of the bifurcation pattern of period $2^n$ just before it gives birth to period $2^{n+1}$ [7, 9]. This theory might have successful impact on our principal study.

**We are now in a position to explain our objective as follows:**

Recent suggestions that speech production may be a **nonlinear** process [7,10 12] have sparked great interest in the area of nonlinear analysis of speech with a number of studies being conducted to investigate the existence of low dimensional chaotic attractor of speech. S. McLaughlin through his paper [12] attempts to show whether speech is chaotic by examining a range of invariant geometric features for a set of prolonged English vowel sounds and the results presented show that low dimensional chaotic attractors do exist in speech. Time series data provides a one dimensional projection of a system's underlying geometrical attractor which can be reprojected back into a higher dimensional state space by means of time delay embedding. If the embedding dimension is high enough then the embedded attractor should have invariant qualities that can be quantified and analysed to reveal its structure. In his paper he makes use of the correlation dimension [12], which measures the space filling qualities of the attractor, and the Lyapunov spectrum [7,8] which measures the attractor's sensitivity to initial conditions. The speech data is embedded into state space using an optimal time lag which is found by analysis of the mutual information, and the correlation dimension is evaluated to provide an indication of the number of Lyapunov exponents that should be evaluated to avoid spurious readings. The resulting Lyapunov spectra reveal the existence of positive exponents which serve as an indication of chaos and a limitation on predictability. It is also shown that the correlation dimension of vowels may be linked to the manner of articulation; and further, he suggests that vowels produced with high articulation have lower dimensionality than those produced by low articulation and that speech is highly chaotic.

Moreover, they suggest that Formant analysis forms the basis for one of the most successful models of speech which is the formant model. This separates the vocal folds and the vocal tract into a source and a filter which are independent of each other. The source, representing the vocal folds impulse, is modelled by a repeated impulse which may be filtered to produce a non-flat spectral distribution and the vocal tract is modelled by a number of bandpass filters centred on the formant frequencies. Their initial model assumes that the system is linear which has been shown in a number of papers not to be the case. In particular, works by Teager [14] and Casdagli [6] have both shown that speech is a nonlinear process and that the effects of the nonlinearities are not negligible.

Tokuda et al. [15] have an on-going study of Japanese vowels. Their initial work focused solely on the Japanese vowel /a/, and led to the tentative conclusion that this vowel does display chaotic behaviour. They make use of principle component analysis (PCA) to decide on a suitable embedding dimension, which they found to be three. They claim to have observed stretching, folding and compression of the state space reconstruction by examination of Poincar’e sections, which would indicate chaos. This is
further quantified by their finding of a positive–zero–negative Lyapunov spectrum. Finally, they calculate the fractal dimension of speech [10, 11, 13] (using the Lyapunov dimension), which is found to be between two and three. However, as they indicate, the Lyapunov results should be treated with caution, due to possible difficulties in the calculation. In particular, it would seem that the use of the Sano and Sawada algorithm [4, 9, 15] may give misleading results on such real world data, as it will be adversely affected by noise. They have searched for deterministic nonlinear structure in the pitch–to–pitch variations of vowels with the conclusion that these variations may be fractal.

Vocal fold is a non-linear system in which vibrating, region of different vibratory regions may overlap. The overlapping of the vibratory regions causes a serious concern for the control of the vocal fold. Due to this overlapping, same vocal chord may execute more than one vibration pattern and there may be even spontaneous change from one vibration pattern to another. The sudden qualitative change in the vocal fold vibratory pattern may be attributed to as being induced by long pressure, tension in the vocal fold, and asymmetry. This characteristics feature of the vocal fold is called bifurcation [6,7,8]. There are evidences that vocal folds vibrate due to combine efforts of aerodynamic forces and the elastic properties of the vocal fold. The train of discrete air puffs generated due to opening and closure of the glottis are filtered and transformed into voiced speech by the vocal tract. The different nonlinearities developed during the vocal fold vibration are:

(a) Non-linear interaction of the air flow and glottal area [2,4,6]

(b) Non-linear stress-strain characteristics of the vocal fold [13], and

(c) A restoring force arising due to collision of vocal folds [12].

In view of these non-linear features of the vocal folds, it has been reported by many workers [4, 5,8,9] that complex bifurcation pattern and low dimensional chaos are the obvious phenomena during the phonation. Herzel, H and Knudsen, C [11] reported that the feedback from the vocal tract to the glottis, which usually not considered in any vocalization model, plays important role in the non-linear feature of the vocal fold. The sub-harmonic and chaos are reported as two basic effects found associated co-existed in the cries of healthy infants and infants with several perinatal complications [8]. However the occurrence frequency and the duration of the chaotic episode are holding diagnostic relevance. The chaos is less frequent in case of adult speakers. The muscular action of the vocal fold is mainly responsible for the measure of the effective length of the vocal fold, its mass and tension in the fold. The vibrations of these vocal fold parameters are usually slower than the vocal fold vibration during phonation onset. During the phonation, a diot of parameters can induce qualitative changes in the vocal fold dynamics which are sudden and irregular. These sudden changes are related to bifurcation of the subsequent dynamical system [5, 13]. It has been reported that during phonation there is a coexistence of a chaotic attractor, a steady state and a large limit cycle with the collisions of the vocal folds. The coexistence of different attractors is possible source of non-linearity manifesting the chaotic episode. Herzel, H and Knudsen, C [11] with the help of a symmetric two-mass model of vocal fold vibrations reported that the vibrations may be more complex even during the strongly simplified phonation. It is further reported that the variation of the vocal fold is complicated in case the vocal folds are slack and thick.
which has been subsequently supplemented by the low Q-values in the Ishizaka-Flanagan model i.e. at the lower range of the fundamental frequency [5,14].

In the vocal fold, which is a non-linear system, different vibratory regimes may overlap, which subsequently give rise to more than one vibration pattern for the same vocal fold. In these overlap regimes; involuntary, spontaneous jumps may occur from one vibration pattern to another. This causes the subsequent occurrences of the bifurcations. Though with the help voice range profile (VRP) it is reported possible to map out the region in which phonation is possible, and the central focus of the VRP is the Hopf-bifurcation [6,7,8] i.e. the bifurcation associated with the phonation onset, but in the entire bifurcation scenario due to VRP, particular voice type or vocal fold configuration while generating the VRP.

Many workers in the field of speech analysis, synthesis and recognition often used a concept of resistors to describe different vibratory regimes of the vocal fold [6, 9,11] configuration. Typically, these registers include -vocal fry or pulse register, chest, falsetto or head, whistle, flageolet, or flute register. Berry et al [50] conducting an experiment on the languages of five large mongrel dogs (killed), found for the rounded spectra that these were spontaneous jumps between chest-like and falsetto-like phonation and a small variation in the elongation asymmetry 1mm increase while jumps to flute-like phonation. They further reported large amplitude vibration & intense his/her harmonics for chest-like phonation. The falsetto-like phonation was characterized by small amplitude, incomplete closer, weak phase delay between upper and the lower edges of fold, less intense his/her harmonics. They flute-like phonation exhibits newly the fry of the previous two, very weak harmonics. They also reported the vorticity at the vocal fold fry.

It has been reported [5,11] the chest-like vibration of the larynx configuration co-existed with irregular vibrations. The period, as reported, is 1sec from the chest-like vibration to irregular vibration. Berry et al [3a] further reported a period doubling fourth transition from the chest-like vibration of the larynx the irregular vibration. As one of the characteristic features of the existence of chaos is the period doubling, so the transition of the chest-like vibration of the larynx to the irregular vibration with period doubling as reported by Berry et al could be thought as one of the real ground to advocating the irregular oscillation associated with chaos.

2. Bifurcation and Chaos on Nonlinear models: The Main Study

We know that formant frequency spectra and cepstral coefficients can be approximately represented by some nonlinear models of the form, [3,4,8,10] :

\[ Y = b_0 + b_1 x + b_2 x^2 + b_3 x^3 \]

with variation of the coefficients . To make rigorous study of the existence of bifurcations and chaos of these types of models, we attempt to give comprehensive idea with the help of the Logistic model [7, 8] :

The Logistic Model is :

\[ f(x) = \lambda x (1 - x), \quad 0 < \lambda \leq 4, \quad 0 \leq x \leq 1 \]

where \( \lambda \) is an adjustable parameter.

Firstly, we need to explain Feigenbaum universality.

**The Feigenbaum Universality** [7] , We present the Feigenbaum universality with the aid of some results of a numerical investigation on the Logistic map:

\[ f(x) = \lambda x (1 - x), \quad 0 < \lambda \leq 4, \quad 0 \leq x \leq 1 \]

If \( \lambda \neq 0 \) and \( x > 1 \) or \( x < 0 \), then \( f(x) < 0 \), so let us take \( 0 \leq x \leq 1 \) and
\[ \lambda \leq 4 \], in order that \( f \) maps the interval \([0, 1]\) into itself. The logistic map \( f(x) = \lambda x (1 - x) \) has two fixed points \( x = 0 \) and \( x = 1 - 1/\lambda \). These two fixed points are the intersection of the line \( y = x \) and the graph of the function as shown in Fig 6.1. We evaluate \( df/dx \) at each of these fixed points. We also know that if the absolute value of the derivative at a fixed point is less than 1, then it is a stable fixed point. If it is greater than 1, it is an unstable fixed point. If it is exactly equal to 1, the bifurcation occurs. Now the derivatives are:

\[
\frac{df}{dx} \bigg|_{x=0} = \lambda \quad \text{and} \quad \frac{df}{dx} \bigg|_{x=1-1/\lambda} = 2 - \lambda. \tag{1}
\]

We see from the above that the fixed point \( x = 0 \) is an attracting (stable) fixed point for \( \lambda < 1 \) and that the fixed point \( x = 1 - 1/\lambda \) is a repelling (unstable) fixed point for \( \lambda < 1 \). Thus for \( \lambda < 1 \), we expect all trajectories with \( 0 < x < 1 \) to approach the fixed point \( x = 0 \) (except for the trajectory that starts exactly at \( x = 1 - 1/\lambda \)). We also see from equation (1) that for \( 1 < \lambda < 3 \) the two fixed points exchange stability. That is, for \( \lambda > 1 \), the fixed point \( x = 0 \) is an unstable fixed point. For \( 1 < \lambda < 3 \), the fixed point \( x = 1 - 1/\lambda \) becomes a stable fixed point.

Having studied the dynamics of the quadratic iterator \( f \), in detail for parameters between 1 and 3, we continue to increase \( \lambda \) beyond 3. For such large parameters the fixed point \( x = 1 - 1/\lambda \) is not stable anymore, it is a repeller. Hence, the first bifurcation value is \( \lambda_1 = 3 \).

Next, let us consider the iterated map \( f^2(x) \). The fixed points of \( f^2 \) are given by the equation \( f^2(x) = x \). This gives

\[-\lambda^3 x^4 + 2 \lambda^3 x^3 - (\lambda^3 + \lambda^2) x^2 + (\lambda^2 - 1) x = 0\]

The roots of the above equation are: \( 0, 1 - 1/\lambda, \)

\[ x_{11}^* = (\lambda + 1 + \sqrt{\lambda^2 - 2 \lambda - 3}) / 2 \lambda \]
and \( x_{12}^* = (\lambda + 1 - \sqrt{\left( \lambda^2 - 2 \lambda - 3 \right)}) / 2 \lambda \).

These four points are the intersections of the graph of the function \( f^2(x) \) and the equilibrium line \( y = x \) as shown in Fig. 2.

Fig 2 Graphs of \( f^2(x) \) and \( y = x \); Intersection points (\( \bullet \)) are fixed points.

Considering only parameters between 1 and 4, we note that these solutions are defined only for \( \lambda \geq 3 \). The new two fixed points \( x_{11}^* \) and \( x_{12}^* \) are attracting stable fixed points of a “two-cycle” within the range \( 3 < \lambda < 1 + \sqrt{6} \). The two-cycle points \( x_{11}^* \) and \( x_{12}^* \) are fixed points of the second iterate function \( f^2(x) \).

Now if we take \( \lambda > 1 + \sqrt{6} \), these two points will turn to be unstable. This means that \( \lambda_2 = 1 + \sqrt{6} \) is the second bifurcation point. Thirdly, we consider the
Fig 3 Top and bottom diagrams are respectively the graphs of \( f^4(x) \) and \( y = x \), and \( f^8(x) \) and \( y = x \). Fourth iterated map \( f^4(x) \) and the line \( y = x \) as shown in top Fig 6.3. We repeat the same analysis and have found that these four points remain stable in the range \( 1 + \sqrt{6} < \lambda < 3.544090... \) and will be unstable otherwise. Thus \( \lambda_3 = 3.544090... \) is the third bifurcation point. In the same way, considering 8th, 16th, 32th, .......
iterations of the map, we can find 4th, 5th, 6th, ...........bifurcation values (with the help of a computer programme) as 
\( \lambda_4 = 3.568759 \ldots \), \( \lambda_5 = 3.569692 \ldots \), \( \lambda_6 = 3.569891 \ldots \) etc. Based on these values, we compute:

\[ \delta_1 = (\lambda_2 - \lambda_1)/ (\lambda_3 - \lambda_2) = 4.7514 \ldots, \]
\[ \delta_2 = (\lambda_3 - \lambda_2)/ (\lambda_4 - \lambda_3) = 4.6562 \ldots, \]
\[ \delta_3 = (\lambda_4 - \lambda_3)/ (\lambda_5 - \lambda_4) = 4.6682 \ldots, \]
\[ \delta_4 = (\lambda_5 - \lambda_4)/ (\lambda_6 - \lambda_5) = 4.6687 \ldots \]
and so on. Then the Feigenbaum constant is evaluated as 
\[ \delta = \lim_{k \to \infty} \delta_k = 4.66920 \ldots \]

The nature of \( \delta \) is universal i.e. it is the same for a wide range of different iterators. The surprising result is that this regular phenomenon can be viewed upto the value \( \lambda = 3.572 \ldots \) and highly irregular behaviour can be observed in the range \( 3.572 \ldots < \lambda \leq 4 \). Fig 5, below. This region is chaotic and a challenging topic for research. This theory has shown a close link between a regular system and a chaotic system.
Fig 4 A typical bifurcation diagram for the Logistic map
Fig 5: Several regular bifurcations finally lead to the chaotic region (shaded portion).

Now our question is: Can we develop this type of bifurcation theory in our study of Assamese, Bodo and Rabha vowels? Our success is at an infant stage, however we have opened up further research in this direction.

3: Bifurcation and Chaos as observed in Assamese, Bodo and Rabha Phonemes spectras:

One of the interesting feature that have been observed in the present study, as depicted in Fig.-A, Fig.-B, Fig.-C and Fig.-D for Assamese vowels, and Fig.-E and Fig.-F for Rabha vowels, there is an intermittent jumps between two adjacent irregular vibrations, which last, for the fraction of a second. Further, from the spectrograms as depicted in the figures, it is clear that irregular vibration is totally random, as opposed to the report of David A.Berry et al [3a]. The effect of irregular vibrations seems more prominent towards the higher end of the formant frequencies. The splitting of the spectra into two components are also some uncommon features of the spectra as obtained in the present study.

More generally, the irregular spectra of Assamese and Rabha vowels, corresponding to female speakers can be summarized as:-

The spectra's onset is always associated with a random noise followed by intermittent jumps and finally, followed by formant or harmonic splitting. Such a feature is found common in most of the female speakers. However, the Bodo vowels spectra do not reveal any such type of random or irregular characteristics.

Such a feature can be resembled with the bifurcation and chaotic behaviors of the speech signal. As reported by the earlier workers that the onset of bifurcation is attributed to a period-doubling which is known as the common precursors of chaos, but the present
study does not reveal any such circumstances. This is clearly evident from the spectrograms. Thus, the obvious chaotic behavior of the speech spectra as obtained in the present study could not be attributed to the effect of period doubling.

4: Observations and Results:

The present study, however, reveals the fact that there is considerable overlapping of the region of single vocal fold vibrations with chest like and irregular vibrations. A transition from irregular vibration to chest like vibration, Fig.A and Fig.C occurs within the range of about 0.05sec to 0.15sec as manifested by sudden intermittent jump.

Another interesting feature of the speech spectra, as obtained in the present study, is the co-existence of two independent frequencies, demonstrated by the two sets of spectral lines, Fig. A and Fig. C. This situation may be thought as the onset of bifurcation.

Thus, it is concluded in the present study of bifurcation & chaos in the vowels' spectra, that, (i) the instability of the speech spectra, (ii) the sudden intermittent jump of the formants as and when transition occur from the random irregular vibration to the chest like vibration, (iii) the overlapping of the irregular vibration and chest like vibration and finally (iv) the co-existence of two frequency components in the spectra, all reveal the onset of bifurcations and chaos associated with the utterances of most of the Assamese and Rabha speakers. Such a feature is not clearly observable in case of Bodo speakers.

Thus, the Assamese and Rabha speakers are found more obvious source of generating the bifurcation and chaos, which must be due to elongation asymmetry of their vocal fold dynamics. However, the mode of articulation and the tonal features of the language may also be accounted for while considering effectiveness of the chaos and bifurcation in the language.
Fig. A. Top and bottom figures show bifurcations of Assamese vowels (male).
Fig. 3. Top and bottom figures show bifurcations of Assamese vowels (male)
Fig. Top and bottom figures show bifurcations of Assamese vowels (Female)
Top and bottom figures show bifurcations of Assamese vowels (Female)
**Fig. 6.1(a): Bifurcation of Rabha vowel /a/ (Female)**

**Fig. 5:** Bifurcation of Rabha vowel /a/ (Female)
Fig. F  Bifurcation of Rabha vowel /a/ (Female)
References:


