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MODELING THE TRANSPORT OF MAGNESIUM IN GROUND WATER INFLUENCED BY LINEAR VELOCITY IN HOMOGENEOUS FORMATION IN DIOBU PORT HARCOURT, RIVERS STATE OF NIGERIA

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ABSTRACT

High rate of water hardness is a serious threat to human utilization. The rate of magnesium in ground water at Diobu Port Harcourt were found to be very high; this implies that shallow aquifers water can not be used for human consumption. Therefore the abstraction of ground water free from magnesium is found to be very hard to construct. The rate of magnesium deposition varies in the study location, this become a serious issue as this has generated life threats from this trace mineral sources. Mathematical model were developed to monitor the trace of magnesium content in the study area, the model were developed considering the formation characteristics that played significant roles on the migration process to ground water aquifers, the model will serve as a base line for ground water monitoring and assessment of magnesium content in the study area. Practicing engineers and scientist will definitely apply these concepts as a bench mark in solving the problem of ground water quality assessment on magnesium content in the study area.

KeyWords

Wastewater, Indus Waters Treaty (IWT) ,National Environmental Quality Standards (NEQS) ,dissolved magnesium

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1. Introduction

Fresh water resources are the most valuable assets of any human civilization. They serve a pivotal role in overall economy of a country due to inevitable demand of water in all sectors of life. Surface water quality is influenced by various natural processes and anthropogenic activities. In many developing countries, wastewater is disposed into the natural water bodies due to their capacity to assimilate and dilute the harmful constituents of the effluents. As municipal and industrial demand for freshwater rises, increasing effluents of low quality are dumped unchecked into the natural water bodies resulting in further degradation of their water quality. Consequently, human health and crop yields are being affected or threatened in many cases. "Each water body can assimilate a certain amount of effluents depending on numerous factors. The water quality management attempts to protect the uses of water bodies facing the threat of pollution". (McBride, 2002; Mohammed et al. 2002; de Azevedo et al. 2000; Somlyódy et al. 1998). Pakistan is one of the world's most arid countries, with an average annual rainfall of under 240 mm. Throughout the history, people of this region have adapted to low and poorly distributed rainfall by depending on an annual influx of about 180 billion cubic meters (BCM), into the Indus river system (including Indus, Jehlum, Chenab, Ravi, Bias and Sutluj rivers). The rivers flowing in Pakistan largely emanate from the neighboring countries and are mostly derived from snowmelt in the Himalayas. Alteration of natural flows of the trans-boundary Rivers, as happened in the subcontinent of Pakistan and India, threatens their water quality. The Governments of Pakistan and India signed Indus Waters Treaty (IWT) in 1960 The rapid industrialization also adversely effects the environment directly and indirectly. Industrial development manifested due to setting up of new industries or expansion of existing industrial establishments results in the generation of industrial effluents. The present method of transportation of these effluents, their ultimate disposal and treatment facilities for making effluents innocuous and safe are inadequate, unplanned and suffers negligence and shortage of funds (Kul-

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karni, 1979). The net result is large scale pollution of the water bodies which may act as a source of water supply for domestic use of inhabitants of localities. This loss of water quality may cause health hazards to human and livestock, death of aquatic life, crop failure and loss of aesthetics. The industrial waste and domestic sewage is dumped through surface ditches (drains) into water bodies of the country. A recently conducted nation-wide wastewater assessment showed that total waste water supply in Pakistan is $4.6x10^6$ m^3/day, and of total 7.85 million m^3/day of wastewater (30 percent of the total) is used for irrigating an area of 32500 hectares. It has also been estimated that 64 percent of the total wastewater is disposed off either into rivers or in Arabian sea. Similarly, 400,000 m^3/day wastewater is additionally added to canals. These practices threaten both human health and the environment at downstream and more importantly reduce the effective availability of Pakistan's already short water supplies (Ensink et al., 2004). Indiscriminate disposal of sewage and industrial effluent has seriously affected the quality of surface water. A recent report of WWF (2007) stated that the quantity as well as quality of water resources of Pakistan is strongly affected by tremendous increase in population, urbanization and unsustainable water consumption. There is very little separation of municipal and industrial effluents in the country. Both the effluents flow directly into the nearby natural water bodies (rivers or canals) through open drains. Unfortunately, no surface water quality standards have been established in Pakistan. National standards are available only for wastewater (industrial and municipal effluents) but these are rarely enforced. In this scenario, the water quality issue in Pakistan has not yet got its due importance. A comprehensive water quality monitoring program is indispensable to assess the water quality status of the national rivers. After the collection of monitoring data on water quality, it is needed to convert it into an understandable format that can be easily interpreted. Another important issue to be addressed is the little attention given to formulate surface water quality standards according to different water uses. National Environmental Quality Standards (NEQS) established in 1993 are available only for municipal and liquid industrial effluents and do not provide any guideline for the receiving water bodies. PSI (1987) and PCRWR (2002) have

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drafted water quality standards for drinking and irrigation waters but their enforcement is still pending.

Quality water is exclusive of doubt a limiting substance, not only for humans but for all life. Yet, as human population, activity, and pollution continue to increase, natural sources of readily useable water are declining at an alarming rate. Humans currently divert or regulate more than half of globally available freshwater runoff for their own purposes (Postel et al., 1996), including the use of large dams or diversions on rivers and the widespread creation of other artificial catchments. Additionally, groundwater sources are becoming increasingly used for agricultural and urban purposes at rates that far exceed the natural ability of these reserves to recharge themselves. As a result of human use and disturbance, water quality continues to degrade through alteration of natural physical conditions and from a variety of pollutants including pesticides, excessive nutrients, pathogenic organisms, and the most ubiquitous item, sediment (U.S. EPA, 1994). These human induced changes to our streams and rivers have serious impacts on both aquatic and terrestrial life. Additionally, artificial linkages of waterways, global transportation and introduction of exotic species have led to unnatural competition, predation, and hybridization between native and non-native species. These impacts pose a major threat to survival of naturally-occurring aquatic organisms and the ecological functions they perform. Recent estimates of vulnerable, imperiled, or extinct aquatic animal species in the United States are approximately *35%* of amphibian and fish species, and over 65%of crayfish and unionid mussel species. These risk estimates are two to four times greater than those for similar groups of terrestrial species (Richter et al., 1997). Modifications to rivers and streams for transport, flood control, or hydropower often result in major alterations of water resources. These activities create changes in temperature, salinity, dissolved gases such as oxygen, and acidity or alkalinity of the water. Industrial water exports may contain toxic metals or chemical compounds, high concentrations of fiber or byproduct materials, and may have substantially different temperature from receiving waters (Cooper et al 1997, 1998. 2000).

2. Theoretical background

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Magnesium is one of the deposited natural in soil and water environment, this type of the minerals were found to deposit soil dissolving with calcium to generate hardness of water, when a water is too hard it will not be of good quality to drink, high rate of magnesium content has been a serious threat to quality water in the study location, because the mineral were found to be more than the limited standard. Shallow aquifers where found to deposit in Diobu, but due to this high concentration of magnesium in the study area deep well are always recommended for quality water, most borehole drilled are not drill to the optimum aquiferous zone, over sixty percent of bore hole drilled contain high concentration of magnesium, most of this water can not be used for thorough domestic use, the trace of high content of magnesium carried out through some water analysis in the study area. High degree of porosity in the formation also played significant role in fast migration of the trace minerals to ground water aquifers, the geological studies explain the predominant of homogeneous formation, this is through alluvium influence, the formation characteristics influence the deposition of these trace minerals to ground water aquifers. The development of mathematical model to monitor the trace of this mineral where imperative, because it will definitely become a base line for practicing engineers and scientist to understand the deposition of this trace minerals thus determined there rate of concentration, the model developed will monitor the deposition of the trace mineral in the study location.

3. Governing equation

$$\frac{Vi\partial^2 C}{\partial t^2} = -\left[\frac{Kj\varepsilon p}{\mu}\right]\left[P\frac{\partial C}{\partial x} + Pg\frac{\partial C}{\partial xi}\right] \qquad \text{.................................} \qquad (1)$$

Taking the Laplace transformation of (1)

The governing equation were established considering the velocity of transport through permeability and porosity influence, this also include fluid pressure on the aquiferious zone. This model consider this type of formation characteristics base on the influence from the formation variables, these were applied to form a system to develop the governing equation. The governing equations were established considering the velocity of transport through the permeability and porosity influences from fluid pressure on aquiferious zone.

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$$\frac{\partial^2 C}{\partial t^2} = SC_{(0)} - SC_{(x)} - C_{(0)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

$$\frac{\partial C}{\partial x} = S^1 C_{(x)} - SC_{(x)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

$$C_{(x)} = C_{(0)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

Substituting equation (2), (3) and (4) into equation (1) yields

$$Vi\left[S^2 C_{(x)} - SC_{(x)} + C_{(0)}\right] + \frac{Kj\varepsilon p}{\mu} + \left[PSC_{(x)} \_ Pg\, SC_{(x)}\right]C_{(0)} \quad \ldots\ldots\ldots\ldots\ldots \quad (5)$$

Applying Laplace transformation, the variables were expressed, where there relationship were critical assessed at different conditions, therefore equation 2-4 were substituted in to equation one to yield equation (5).

$$Vi\ S^2 C_{(t)} - ViS^1 C^1{}_{(t)} + C_{(0)} + \frac{Kj\varepsilon p}{\mu} C_{(0)} + C_{(0)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

Considering the following boundary condition at $t = 0,\ C^1{}_{(0)} = C_0 = 0$ ……… (7)

We have

This condition developed an expression through the Laplace transformation; these were expressed by integrating the variables considering various conditions. This concept will definitely continue to express it function through this these methods applied. Expressing the solution of equation 7 it was necessary to determine the boundary conditions, the condition were determine base of the considered variables in developing the governing equation. Boundary condition were integrated into the expression to produce the behavior of the contaminant, since it is not a living organism, the transport of dissolved magnesium are influence by the geological formation in the study area.

$$C_{(t)}\left(Vi\ S^2 - Vs + \frac{Kj\varepsilon p}{\mu} + P + Pg\right) = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

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Linear velocity were considered in equation 8 where the expression where assumed to be equal to zero, this condition where assumed when

$$C_{(t)} \neq 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (9)$$

But considering the boundary condition

At $t > 0,\ C^{1}{}_{(0)} = C_{(0)} = C_o$ ……………………………… (10)

$$S^2 C_{(t)} - \frac{Kj\varepsilon p}{\mu} SC_{(x)} + Ps\,S\,C_{(x)} = Vi\,Sc_o + Vi\,C_o + \frac{Kj\varepsilon p}{\mu} C_o + Pg\,C_o \qquad \ldots\ldots \qquad (11)$$

This developed to another boundary conditions at equation (10) were time assumed to be greater than zero with respect to concentration. The expression were integrate in the boundary conditions as expressed above, thus considering linear velocity under the influence of permeability of the formation including the rate of porosity, this parameters can not be in existence without the fluid pressure as it is expressed in equation, 11 the boundary values were integrated as expressed in the equation, it consider magnesium content with respect to time of transport at various formation, the behaviors of the trace minerals where expressed in equation 12 considering high content of magnesium with respect to time independent, such condition generated the expressed equation as it relate with the formations influence that allowed the concentration to migrate from one region to another.

$$\left[Vi\ S^2 - \frac{Kj\varepsilon p}{\mu} + P + Pg\right] C_{(t)} = \left[Vi\ S + Vi + \frac{Kj\varepsilon p}{\mu} + P + Pg\right] C_o \qquad \ldots\ldots\ldots\ldots \qquad (12)$$

$$C_{(x)} = \frac{Vi\ S - ViS + \frac{Kj\varepsilon p}{\mu} + P + Pg}{Vi\ S^2 - \frac{Kj\varepsilon p}{\mu} + P + Pg} C_o \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (13)$$

Porosity of the soil influences the fluid pressure at various aquiferous zones, allowing high concentration to migrate by maintaining the plug flow law as a system. The established re-

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lation of all the variables considered the migration with change in distance, these conditions were expressed in equation 13, the boundary values were integrated as expressed in equation 13. Further expressed solution were also necessary, because the relation between the variables can not be expressed thoroughly with Laplace application, therefore quadratic method were integrated, the variables were expressed to obey the rule of quadratic functions considering the behavior of the traced mineral in the system.

Applying quadratic equation, we have

$$S = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (14)$$

Where $a = Vi$, $b = \frac{Kj\varepsilon p}{\mu}$, $c = PPg$

$$S = \frac{\frac{Kj\varepsilon p}{\mu} \pm \sqrt{\frac{Kj\varepsilon p}{\mu}^2 + 4ViPPg}}{2Vi} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (15)$$

$$S_1 = \frac{\frac{Kj\varepsilon p}{\mu} - \sqrt{\frac{Kj\varepsilon p}{\mu}^2 + 4ViPPg}}{2Vi} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (16)$$

$$S_2 = \frac{\frac{Kj\varepsilon p}{\mu} + \sqrt{\frac{Kj\varepsilon p}{\mu}^2 + 4ViPPg}}{2Vi} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (17)$$

$$S_1 = \frac{\frac{Kj\varepsilon p}{\mu} + \left[\sqrt{\frac{Kj\varepsilon p}{\mu}^2 + 4ViPPg}\right]}{2Vi} S_2 + \frac{\frac{Kj\varepsilon p}{\mu} - \left[\sqrt{\frac{Kj\varepsilon p}{\mu}^2 + 4ViPPg}\right]}{2Vi} \ell^{\frac{\frac{Kj\varepsilon p}{\mu}\left[\sqrt{\frac{Kj\varepsilon p}{\mu}^2 + 4ViPPg}\right]}{2Vi}\frac{L}{v}} +$$

$$\frac{\left[-\frac{Kj\varepsilon p}{\mu} - \frac{Kj\varepsilon p}{\mu}\sqrt{\frac{Kj\varepsilon p}{\mu}^2 + 4ViPPg}\right]}{2Vi} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (18)$$

Applying Laplace inverse of the equation, we obtain

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$$C_{(t)} = \left[\frac{Vi}{t} + Vi + \frac{Kj\varepsilon p}{\mu} + P + Pg\right] C_o \ell^{\left[\frac{\frac{Kj\varepsilon p}{\mu}\sqrt{\frac{Kj\varepsilon p}{\mu}^2 + 4ViPPg}}{2Vi}\right]^t} + \ell^{\left[\frac{\frac{Kj\varepsilon p}{\mu}\sqrt{\frac{Kj\varepsilon p}{\mu}^2 + 4ViPPg}}{2Vi}\right]^t} \quad \ldots\ldots\ldots \quad (19)$$

But if $t = \frac{d}{v}$

$$\left[C\,[d,v] = Vi\Big/\tfrac{d}{v} + Vi + \frac{Kj\varepsilon p}{\mu} + P + Pg\right] C_o \ell^{\left[\frac{\frac{Kj\varepsilon p}{\mu} + \sqrt{\frac{Kj\varepsilon p}{\mu}^2 + 4ViPPg}}{2Vi}\right]^{\frac{d}{v}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (20)$$

Considering the following boundary conditions at

$$t = 0,\ C^1 = 0,\ C = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (21)$$

The expressed solution where applied from equation 14 to 18 where there was an interaction of all the variables to determine there various function in the system, the developed expression proceed subjected to the expressed equation, an inverse expressed equation were applied as the given solution where expressed in equation 20, the condition were expressed considering when the trace mineral reverse back. The influence of the formation characteristics, and the dynamic of the system were considered as expressed in equation 20 and 21.

$$C_{(x)} = \left[\frac{Vi}{t} + Vi + \frac{Kj\varepsilon p}{\mu} + P + Pg\right] C_o \left[\ell^{\frac{\left[\frac{Kj\varepsilon p}{\mu}\sqrt{\frac{Kj\varepsilon p}{\mu}^2 + 4ViPPg}\right]}{2Vi}}\right]^{\frac{d}{v}} + \left[\frac{\left[\frac{Kj\varepsilon p}{\mu}\sqrt{\frac{Kj\varepsilon p}{\mu}^2 + 4ViPPg}\right]}{2Vi}\right]^{\frac{d}{v}} \quad \ldots \quad (22)$$

At $C^1{}_o = t \neq 0$

Again $C^1{}_{(o)} = C_{(o)}$ so that $C_o = Vi + \frac{Kj\varepsilon p}{\mu} + P + Pg\ C_o = [1+1]\ i.e.\ 0 = \left[0 + \frac{Kj\varepsilon p}{\mu} + PPg\right]^2$

……………… (23)

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Boundary conditions where also expressed considering present state of the traced mineral, the boundary values were integrated in equation 23 and the expression consider the initial concentration under the influence of variation in permeability and fluid pressure in the system.

$$\Rightarrow \quad \frac{Kj\varepsilon p}{\mu} + \frac{Kj\varepsilon p}{\mu} = 0 \qquad \text{..................} \quad (24)$$

So that we have

$$C_{(x)} = \left[2 \tfrac{Vi}{t}\right] C_o \, \ell^{\frac{\left[\frac{Kj\varepsilon p}{\mu} + \sqrt{\frac{Kj\varepsilon p}{\mu}^{2} + 4ViPPg}\right]^{\frac{d}{v}}}{2Vi}} + \frac{\left[\frac{Kj\varepsilon p}{\mu} + \sqrt{\frac{Kj\varepsilon p}{\mu}^{2} + 4ViPPg}\right]^{\frac{d}{v}}}{2Vi} \qquad \text{....................} \quad (25)$$

However, $e^{x} + e^{-x} = 2Cos\, x$ therefore, we have

$$C_{(x)} = \left[2 \tfrac{Vi}{t}\right] C_o \, Cos^{\frac{\left[\frac{Kj\varepsilon p}{\mu} + \sqrt{\frac{Kj\varepsilon p}{\mu}^{2} + 4ViPPg}\right]^{\frac{d}{v}}}{2Vi}} \qquad \text{.....................} \quad (26)$$

Finally the equation 24 to 25 yields the final derived model that expressed all the variables of the system, the expressed model will monitor the variation at various period and depths of the traced mineral, considering the dynamic in concentration influence by formation variables and the activities of man made.

4. Conclusion

Mathematical model of dissolved magnesium in homogenous formation influenced by linear velocity has been thoroughly assessed. The model were found necessary to developed as traced content of magnesium are deposited in the study area, this problem causes hardness of ground water for drinking and other utilization. The rate of magnesium content were very high in ground water, this is through high degree of porosity and the homogenous nature of the formation, these were found to generate high rate of hydraulic conduc-

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tivity of the soil, the geomophological and geochemistry in nature also played major roles in the deposition of high concentration of magnesium in the study area, the model develop model will definitely monitor the rate of magnesium content at various depths in the study area, it can be applied to asses ground water quality and monitoring of traced deposited minerals from magnesium in the study location

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